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Trustee Asset Management Elections: Portfolio Performance Evaluation and Preferencing Criteria

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This article presents a discussion of how trustees can more effectively (1) use performance evaluation measures to discharge successfully the duty of prudent selection, monitoring and retention of investments in financial securities, and (2) employ preferencing criteria appropriate to the unique goals and circumstances of the trust.

The distinction between a portfolio performance evaluation measure and a portfolio preferencing criterion is important to trustees charged with investment and management of trust-owned assets "…in light of the purposes, terms, distribution requirements, and other circumstances of the trust."[1] A performance evaluation measure generally quantifies the reward an investor received for holding a portfolio throughout the evaluation period. Performance evaluation measures usually incorporate both reward and risk where risk measurement is either absolute (e.g., standard deviation or range) or relative (e.g., to a comparative benchmark such as the S&P 500 index). However, risk-adjusted performance measures are not default standards either for determining or for justifying trust investment policy. Commonly used risk-adjusted performance measures provide insight into how efficiently a portfolio generated historical return, given its risk. A risk efficiency

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measure, however, provides little insight into how well the trust is meeting threshold return requirements or into the amount of risk required to generate targeted return. In short, a performance measure may be a preferencing criterion only under a limited set of circumstances.

This article presents a two-part discussion of how trustees can more effectively (1) use performance evaluation measures to discharge successfully the duty of prudent selection, monitoring and retention of investments in financial securities,[2] and (2) employ preferencing criteria appropriate to the unique goals and circumstances of the trust. Part One introduces the Sharpe Ratio performance evaluation metric. Some money managers and financial product vendors may imply that trustees should prefer their products and programs because they exhibit high Sharpe Ratio values. However, the Sharpe Ratio presupposes a variety of simplifying assumptions regarding stock and bond return series. Generally, the Sharpe Ratio is an allowable performance measure only after adjustments to reflect specific statistical characteristics of the time series. Part One continues with a discussion of how trustees can spot abuses of the Sharpe Ratio in performance evaluations of trust portfolios under their administration. This discussion should help trustees become more critical consumers of investment information. Given that investment preferencing criteria are positive in return and negative in risk, Part One discusses the importance of using an appropriate risk model when designing a portfolio's asset allocation. It concludes with a discussion of how trustees can obtain more credible appraisals of portfolio risk as well as more defensible estimates of a trust's required return.

Part Two, to appear in a future issue of *The Banking Law Journal*, refocuses on the trustee's duty to match the investment portfolio to the purposes, terms, distribution requirements and other circumstances of the trust. Although a high performance ratio value may be a legitimate preferencing criterion for the selection and retention of certain investment products, the trustee must also consider a variety of other preferencing criteria including investment liquidity, tax characteristics, prudent risk bounds, ease of marketability in the face of cash flow demands, and so forth. Many of these factors are subsumed in concept of Utility which is a general measure of an investor's satisfaction with a portfolio. Utility, as the term is used both in classical economics and in this essay, is primarily a function of the investor's tolerance for risk. Investor's exhibit heterogeneous utility of wealth functions and, therefore, will demand different portfolios (and different methods of managing portfolios) given their sensitivity to changes in wealth. The article considers ways in which the trustee can match the trust's risk tolerance to the economic objectives of the settlor and to the legitimate needs and expectations of one or more beneficiary classes. Specifically, it explores the interrelationships among utility, asset allocation choices, and wealth management strategies. The discussion considers a variety of preferencing criteria including state preference theory, stochastic dominance, and shortfall probability analysis for dynamic asset management under conditions of cash flows. It concludes by providing an example of how trustees can employ modern econometric tools to facilitate portfolio design and trust administration.

BACKGROUND

This article explores trustee investment preferencing criteria that are primarily quantitative in nature. Such preferencing criteria differ, for example, from general qualitative judgments such as giving preference to mutual funds with a five-star rating from Morningstar or to securities with a strong buy recommendation from a Wall Street broker/dealer. A bright line between qualitative and quantitative analysis is difficult to draw because rating service rankings and analyst opinions derive, in part, from careful consideration of data that may be quantitative in nature. Nevertheless, opinions regarding either an investment's future desirability or past performance are generally written for a representative or average investor, and may not constitute an adequate basis for forming a portfolio well suited to the purposes, terms, distribution requirements, and other circumstances of a particular trust. In certain cases, inadequate attention to quantitative preferencing criteria may lead to a breach of fiduciary duty. It is particularly important for an individual trustee to specify the trust's required return and risk bounds with great precision lest unique preferences and constraints appropriate for personal financial objectives supersede those appropriate for the trust.[3]

Failure to specify required return and acceptable risk bounds can lead to fiduciary surcharge litigation. Consider the following example: A representative from a corporate trust department forms portfolios of securities based primarily on the security research department's opinion regarding the likelihood of above-average performance over the forthcoming period. Although academic studies that suggest a portfolio of 20 to 30 randomly selected securities is sufficient for portfolio diversification within a specific asset class, nevertheless the security research department selects securities based on its unique view of forthcoming market conditions—hardly a random selection process. Needless to say, if the forecasted view is incorrect, the deleterious consequences reverberate throughout the portfolio. If the trustee selects individual securities under a maximize-return-for-the-forthcoming-period criterion, an incorrect economic forecast can result in catastrophic loss. Plaintiff arguments advance the thesis that the corporate trust department was not engaged to form portfolios based on a maximum profit likelihood metric (a 'P&L' preferencing criteria that serves the trust department's interest in collecting fees), but based on the Restatement §90's Prudent Investor Rule.[4]

Although there have been numerous articles discussing the predictive value of qualitative judgments with respect to future investment performance,[5] there are fewer articles that provide guidance on how a trustee can use quantitative analysis as a basis for forming portfolios well suited to a particular trust's return requirements and risk constraints.[6] Most of the extant literature limits itself to discussions of mean-variance optimization techniques which are designed to create "efficient portfolios" but not necessarily prudent portfolios where prudence is defined according to the Prudent Investor Rule's black-letter-law criteria. By contrast, this article begins with a discussion of the Sharpe Ratio which is a measure of how efficiently a portfolio uses risk. The higher the ratio's value, all else equal, the more return-per-unit-of-risk the investment process produces. This ratio is in common usage throughout the investment community and often serves as both a performance evaluation measure as well as a portfolio preferencing measure. Using the Sharpe Ratio as a performance evaluation measure, under a set of highly restrictive conditions, may be valid; however, trustees may go wildly off track if they use it as a preferencing measure as well. The discussion proceeds from a reward-to-risk metric in Part One to a utility-of-wealth metric in Part Two. Familiarity with the concept of Utility allows the trustee to begin to formulate a prudent process directly linking wealth management to specific trust goals, preferences, and constraints. Dollar-wealth maximization strategies are generally not utility-of-wealth maximization strategies because of their attendant risks and costs. As a general principle, a solid understanding of Utility enhances the trustee's ability to calibrate investment decisions to the trust's unique financial needs and objectives.

It was not until the promulgation of Restatement Third and the adoption of state Uniform Prudent Investor statutes that trustee investment practices were brought into greater conformity with modern academic thinking. The motivation to restate trust law reflects, in part, advances in performance evaluation methodology. Modern Portfolio Theory shifts attention from an exclusive focus on the observed outcomes ("track records") to more sophisticated criteria. Prior to the early 1960s, investment performance evaluation focused primarily on realized returns. A returns-only based evaluation is comparable to an investment "horse race" in which the winner is the manager that earns the highest rate of return. Consider the following stylized example of a one-dimensional, returns-based performance evaluation:

Money Manager A: 100 Percent Rate of Return

Money Manager B: 20 Percent Rate of Return

Money Manager C: 10 Percent Rate of Return

An appraisal based solely on realized results indicates that manager A has achieved the "best" performance track record. However, if the trustee is faced with the decision as to which money manager to hire for the forthcoming period, prudence suggests examining how each manager generates his or her returns. A track record reveals only the outcome of an underlying investment process. Outcomes are interesting but not always informative. *Prudence requires understanding and evaluating the manager's return generating process and not merely the outcome of the process.*[7]

Trivially, if manager A purchased some winning lotto tickets, it would probably be imprudent to count on good fortune continuing into the forthcoming period. This eliminates the manager with the best ex post realized return. If managers B and C both formed equally weighted portfolios of 10 stocks, the prudent trustee might decompose each manager's return:

Security	Manager B	Manager C
1	2%	8%
2	-5%	10%
3	0%	-2%
4	-9%	16%
5	-24%	1%
6	1%	9%
7	-3%	20%
8	4%	13%
9	-6%	0%
10	240%	22%
	Average = 20%	Average = 10%

Manager B seems not to evidence much skill in selecting profitable stocks on a consistent basis. Indeed, the portfolio's performance is rescued because of a single stock pick (security #10). The track record may be more a product of luck than skill. Manager C, however, selected only one losing stock during the period and appears to have some ability to identify profitable securities. Credible investment due-diligence is more than a mere consideration of track record—a lesson that Madoff's investors paid a high tuition to learn. Financial analysts attempting to reverse engineer Madoff's track record were left scratching their heads in puzzlement. They were unable to accept his outcomes at face value because they could not understand his return generating process.

The great bear market of 1973-74, which encompassed the OPEC oil-recession and the Watergate political crisis, encouraged the investment practitioner community to adopt more appropriate performance measurement and evaluation techniques. A stock broker's opinion regarding the price target of a security was no longer an adequate basis upon which to make prudent investment decisions.[8] The intellectual wellspring for the new metrics was the academic research that we now term Modern Portfolio Theory ("MPT"). Performance evaluation became two dimensional in the sense that MPT formalized a mathematical relationship between returns and risk. A high return is not a "free lunch;" and, MPT provided a methodology under which returns could legitimately be adjusted to reflect risk. The incorporation of risk adjustments changes the dynamic of the discussion from "what did I earn in the period under evaluation" to "what should I have earned given the risk(s) to which the money manager exposed my wealth."[9]

Reflecting on the 2007-2009 global bear market, it is timely to reconsider how trustees design, implement and manage investment portfolios. This topic forms the article's main focus. Although the article divides into two parts, its overall structure is as follows: (1) an introduction to risk-adjusted performance measures with special attention to the Sharpe Ratio; (2) a discussion of the difference between a *performance measure* that applies to the general population of investors, and a *preferencing measure* that applies to a specific investor or trust; (3) Utility, the Sharpe Ratio and economic state preference criteria;[10] (4) recent advancements in investment policy decision making through simulation of credible risk models; and (5) conclusion.

THE SHARPE RATIO

Definition and Derivation

The Sharpe Ratio, named after the 1990 Nobel Prize winning economist William Sharpe, is the grandfather of risk-adjusted performance measures. The ratio measures the tradeoff between historically realized returns and the amount of risk taken to achieve them. Return is defined as "excess return." This is the return in excess of the risk-free rate earned by the portfolio during the applicable horizon. For example, over a specified period, if the portfolio's return is 10 percent while the U.S. T-Bill rate is 4 percent, the excess return is (10 percent – 4 percent), or, 6 percent. The excess return is placed in the ratio's numerator. Risk is defined as the standard deviation of returns, where standard deviation is a common statistical measurement of dispersion of period-by-period returns from the average return throughout the period. To the extent that period-by-period returns vary, the investor faces uncertainty regarding the final results of the investment venture. Thus, the greater the value of the standard deviation statistic, the greater the amount of risk [risk = return uncertainty]. The standard deviation term is placed in the denominator. Thus, the Sharpe Ratio quantitatively defines the return-to-risk tradeoff.

A common mathematical expression for the Sharpe Ratio [SR] is:

$$SR = \mu/\sigma$$

Where,
μ = excess return [(risky asset return - risk free return)]; and,
σ = standard deviation of the risky asset return.

The Sharpe Ratio is directly linked to an analytical expression for the Capital Asset Pricing Model [the Capital Market Line graphed in return/standard deviation space], independently developed in the mid-1960s by William Sharpe, John Lintner, and J. Mossin:[11]

> Expected Return of a Portfolio = Price of Time + (Market Price of Risk) x (Amount of Risk)
>
> Where,
> The *Price of Time* is the Risk Free Rate [R_f];
> The *Market Price of Risk* is the excess return (risk premium of the market)[12] divided by the standard deviation of the market: $[(R_m) - R_f]/\sigma_m$; and,
> The *Amount of Risk* is the standard deviation of the portfolio: σ_p.
> In mathematical terms:
>
> $$\text{Expected Return} = R_f + [(R_m) - R_f] / \sigma_m] \times [\sigma_p]$$

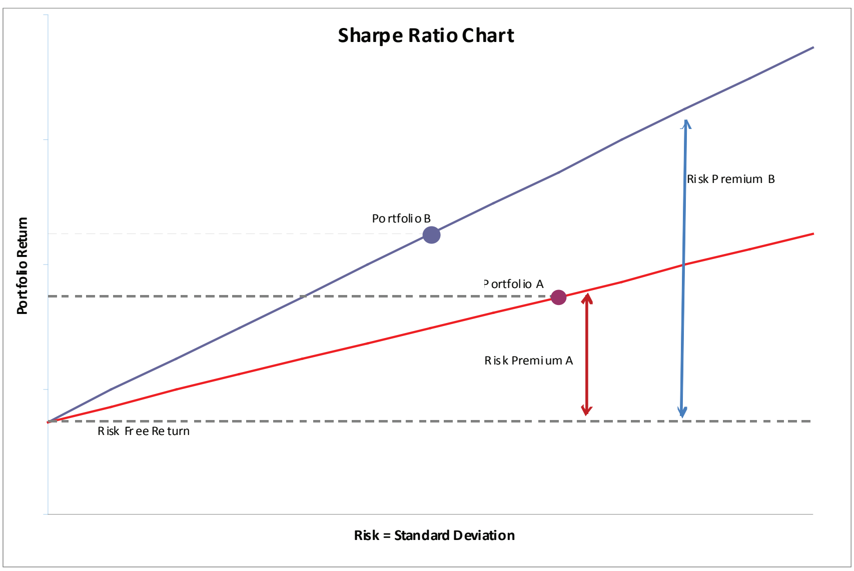
This is the equation of a straight line [y = a + bx] with the intercept defined as the value of the risk free rate ['a' is a constant]; slope 'b' defined as the Market Price of Risk or reward/risk ratio; and 'x' defined as the amount of risk as measured by standard deviation of the portfolio. Thus the Capital Market Line exists in Return/Standard Deviation space. The market-price-of-risk numerator is the *risk premium* $[(R_m) - R_f]$ that the market offers to investors. The denominator is the *amount of risk* (standard deviation) within the market. Thus, the slope of the line determines the additional return needed to compensate the investor for a unit change in risk.[13]

The slope's equation $[(R_m) - R_f] / \sigma_m]$ is the Sharpe Ratio for the broad market. A portfolio that takes more risk than the market should produce a greater return if the investor is to be adequately compensated. A portfolio that takes less risk than the market, however, should not be automatically penalized if it earns less than market return. All investment managers are evaluated on a level playing field because of the Capital Asset Pricing Model ("CAPM") risk adjustment. The Sharpe Ratio is independent of the amount of the investment in the portfolio and is an objective metric upon which to base performance appraisal.

The Geometry of the Sharpe Ratio

The higher the portfolio's slope value (i.e., the greater the value of the Sharpe Ratio), the better the portfolio's risk-adjusted investment performance. This is seen in the following graph which compares the performance of two hypothetical portfolios in risk/return space:

Graph One



The horizontal x axis measures risk (standard deviation of returns) while the vertical y axis measures returns. The dashed horizontal line represents the risk-free rate of return available to investors during the period. A risk-free return, by definition has no variance; and, therefore, its standard deviation (risk) value is zero despite the fact that the dashed line parallels the x axis. The circles, representing the location of portfolios A and B, represent a 100 percent investment in each portfolio. If the investor blends the investment with a short-term T-Bill, the vector of returns moves towards the risk-free rate point-of-origin (the y axis intercept). The intercept is the point where there is a zero-percent investment in the risky portfolio and a 100 percent investment in T-Bills. If the investment is leveraged (i.e., the investor borrows money at the risk-free rate to increase his dollar investment in the risky asset portfolio), the vector of returns moves upwards away from the origin and to the right of the portfolio locations. The distance from the horizontal risk-free return line and the leverage-adjusted investments in portfolios A or B is the excess return or *risk premium* earned by the investor.

The return vectors are linear (straight lines) with a constant slope value. This makes the Sharpe Ratio independent of either the amount of wealth invested or the amount of leverage employed.[14] The Ratio's independence is particularly helpful to trustees seeking to evaluate performance on a level playing field. Clearly, any investor would prefer to have return increase on the vector with the highest slope. This indicates that portfolio B is preferred over portfolio A—that is to say, the Sharpe Ratio of portfolio B is higher than the ratio of portfolio A. In one respect, this is a benefit of the Sharpe Ratio—it is close to a universally applicable measure; but, in another important respect, this is a defect. At any level of wealth, portfolio evaluation with the Sharpe Ratio implies that the preferred portfolio is the one that maximizes the ratio's value. That is to say, the Sharpe Ratio severs the link between the trust's level of current wealth and the risk/reward measurement. Part Two's discussion of investment Utility picks up the threads of this topic and considers the consequences of decoupling unique investor preferences and goals from objective portfolio performance measurement.

Use and Abuse of the Sharpe Ratio: Linear Reward-to-Risk Measure

The Sharpe Ratio's explanatory power and ease of calculation make it a ubiquitous measure of investment performance. As a "back-of-the-envelope" approximation to risk-adjusted returns available in a globally diversified market, it has much to recommend it; and, as a first-order-approximation to portfolio risk, it remains a useful performance metric.[15] As an approximation of portfolio risk, however, there are a few conceptual difficulties:

When portfolio returns are negative (or positive, but below the risk-free rate) the ratio does not permit easy evaluation of results. The problem is illustrated with the following performance statistics:

Investment	**Excess Return (Portfolio Return – T-Bill Return)**	**Standard Deviation**	**Sharpe Ratio (Excess Return ÷ Standard Deviation)**
Portfolio A	-8%	16%	-0.50
Portfolio B	-8%	32%	-0.25

It might seem that the Sharpe Ratio that is *less negative* identifies the best performance over the applicable period. In this example, the trustee may decide (*incorrectly*) to favor Portfolio B despite the fact that it failed to compensate the investor for its obviously greater risk. This does not mean that a negative Sharpe Ratio is devoid of all informational content. A negative ratio value merely indicates that holding a risky portfolio over the period under evaluation was less desirable than holding a risk-free asset. A proposed solution to the negative Sharpe Ratio dilemma is to use absolute value measures; but, these are also subject to interpretive difficulties especially in the face of option-like payoff functions often found in Hedge Fund portfolios.

When portfolios consist of both long and short positions in assets, the Sharpe Ratio can tend towards infinity despite the fact that short positions have payoff functions similar to selling uncovered call options. Theoretically, the risk of writing (selling) uncovered call options is infinite! Several insightful academic articles explain how certain Hedge Fund sponsors can game the Sharpe Ratio. Although the realized returns of some high-risk Hedge Funds have favorable reward to risk tradeoffs when measured by classic MPT metrics, this may merely reflect the fact that embedded risks have not yet manifested themselves in the time series of returns.

Perhaps a more serious defect in the Sharpe Ratio lies in the definition of risk used in the denominator. Many investment risks are non-linear. These include bond default risks, counterparty risks, operational risks, compliance risks, and so forth. The non-linearity of market risk is discussed further in the next section. Assuming, however, that a linear measure of investment risk is reasonable, how well does the Sharpe Ratio capture this risk?

An examination of the following example[16] indicates that defining risk solely in terms of the standard deviation of returns is not always appropriate.

Economic State	**Probability**	**Investment A (Projected Excess Return)**	**Investment B (Projected Excess Return)**
Boom	1/9	+40%	+76%
Normal	7/9	+10%	+10%
Recession	1/9	-20%	-20%

By observation, investment B dominates investment A because, in every possible future economy, B will equal or outperform A.[17] How does the Sharpe Ratio rank the alternatives?

Expected Excess Return A: (1/9)(40%) + (7/9)(10%) + (1/9)(-20%) = 10%
Expected Excess Return B: (1/9)(76%) + (7/9)(10%) + (1/9)(-20%) = 14%

Variance of A Excess Return: $1/9(40\% - 10\%)^2 + 7/9(10\% - 10\%)^2 + 1/9(-20\% - 10\%)^2 = 2.00\%$
Variance of B Excess Return: $1/9(76\% - 10\%)^2 + 7/9(10\% - 10\%)^2 + 1/9(-20\% - 10\%)^2 = 5.84\%$

Standard Deviation of A Excess Return: [Square Root of 2] = .1414
Standard Deviation of B Excess Return: [Square Root of 5.58] = .2416

Sharpe Ratio of Investment A = .10 ÷ .1414 = 0.7072
Sharpe Ratio of Investment B = .14 ÷ .2416 = 0.5795

Decision Rule: Prefer A to B because of the higher positive Sharpe Ratio value.

This is a bizarre result. The Sharpe Ratio metric assigns a higher ranking to the inferior investment simply because risk is defined in terms of the squared deviation from the average. Although investors do not mind returns significantly above the average, nevertheless, the Sharpe Ratio penalizes investment success. How can the use of a ratio that produces such perverse results be justified?

The use of the Ratio is justified under some fairly strict conditions:

- If the distribution of returns is Normal—i.e., shaped like a bell curve—then the Sharpe Ratio is appropriate under the assumption that both risk and return are, in fact, symmetrical. High returns do not come without high risks; and, just because risk has not yet manifested itself does not mean that it is not there. As will be shown, however, the assumption of a symmetrical return distribution is problematic for financial asset returns.
- If the investor cares only about the mean and variance of returns, and, can both borrow and lend at the risk free rate, then the Ratio is a valid performance evaluation metric.

Often, however, the above two conditions do not apply.

Finally, consider two egregious misuses of the Sharpe Ratio. First, it is sometimes used to compare risk-adjusted performance of competing investments over non-synchronous time periods. An investment product vendor may compare a Sharpe Ratio calculated over, for example, a 36 month period, to competing products with Sharpe Ratios calculated over, for example, a 60 month period. Such a comparison, although quantitative in nature, is invalid. Second, Sharpe Ratios should only be used to compare similar investments. If a short-term investment grade bond fund manager earns a lengthy sequence of very small returns in excess of the risk-free rate at a low level of volatility, his Sharpe Ratio value may be much higher than a stock fund manager who earned a far higher return over the period but with a greater risk. One cannot conclude, however, that the bond fund manager evidences greater investment skill. At the limit, small volatility value in the ratio's denominator can distort investment comparisons; and, therefore, caution is recommended when using the Sharpe Ratio to evaluate investment managers. Indeed, there is evidence suggesting that some investment managers deliberately game their Sharpe Ratios in order to win a risk-adjusted performance contest.

Use and Abuse of the Sharpe Ratio: Non-Linear Reward-to-Risk Measure

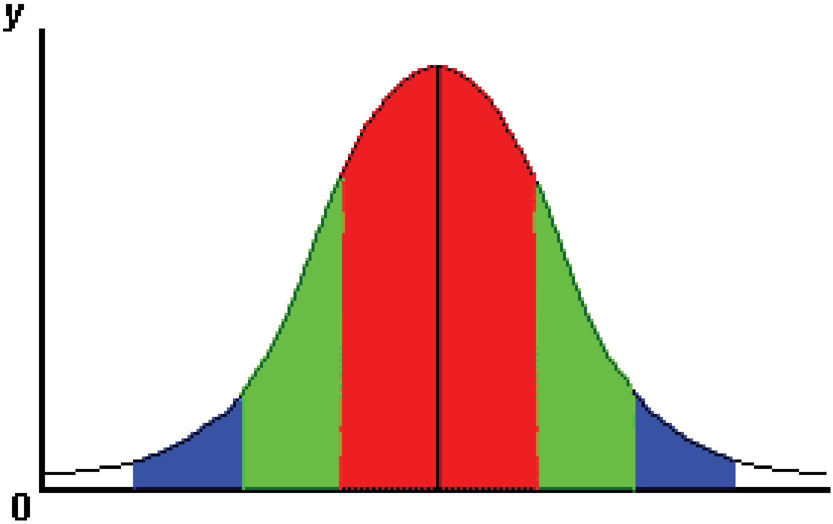
A positive Sharpe Ratio measures the magnitude of the average risk premium relative to the amount of risk taken to achieve it. Perhaps the greatest difficulties with the Sharpe Ratio exist when:

1. Period-to-period portfolio returns are not independent (*autocorrelation*); and,
2. The distribution of the time series of portfolio returns is not normal (*skew and kurtosis*).

Unfortunately, most financial assets manifest various degrees of non-linear risks; and, period-by-period returns are often autocorrelated.

During the early development of Modern Portfolio Theory (approximately 1959 – 1980), limited computing capability (not to mention limited data on financial asset returns) restricted the ability of economists to model complex return distributions. In the face of limited computing power, a log-normal return distribution (the logarithm of period-by-period price change) became an accepted proxy for the empirical distribution. Log-normal returns proved to be both tractable with respect to calculation demands and reasonable as a first approximation to empirical distributions. Models built on log-return assumptions usually guarantee that financial asset returns are stationary (exhibit finite variance). Best of all, the log-normal return distribution is a member of the Gaussian or Normal (bell-curve) distribution family. Characteristics of the Gaussian distribution are well understood by economists; and, as the log-normal distribution became the preferred model, it naturally followed that standard deviation became a preferred risk measurement statistic. This is because, for any Normal distribution, the entire bell curve can be fully characterized by two parameters: mean (average) and standard deviation (the square root of variance of returns away from the average):

Graph Two



In a normal, or "bell-shaped," return distribution, one standard deviation away from the average in either direction on the horizontal axis (the two center sections on the chart above) includes roughly 68 percent of monthly returns. Visually, this means that the two center sections under the bell curve are approximately two/thirds of the curve's total area. Stated otherwise, there is a two/thirds probability that the return in any single period will fall into the two center sections of the distribution. Two standard deviations away from the mean (the two center sections and the areas on either side together) include roughly 95 percent of monthly returns. Three standard deviations account for about 99 percent of monthly returns. Given the symmetrical nature of the distribution, here is how standard deviation can be used as a guide to assessing portfolio risk:

Expected Annual Return of the Portfolio: 8% Standard Deviation of the Portfolio: 16% Expected Range of Annual Returns at a 68% Probability: 8% ±16%(1) = -8% to +24% Expected Range of Annual Returns at a 95% Probability: 8% ±16%(2) = -24% to + 40% Expected Range of Annual Returns at a 99% Probability: 8% ±16%(3) = -40% to +56% Probability of Annual Return less than -40%: ½ of 1% Probability of Annual Return greater than +56%: ½ of 1%.

The Normal distribution is one example of a family of distributions (including Student's *t* and Cauchy distributions) that are symmetric.

The Binomial distribution is related to the Normal distribution and, as the number of trials becomes large, converges to the Normal distribution. The Binomial distribution is familiar because it describes the results of an independent process like coin flips. The probability of flipping heads after a realization of 10 consecutive tails remains, for a fair coin, 50/50. The sequence has no "memory." Each realization in the sequence is independent of all preceding results. However, the time series of financial asset returns often have "memory." Volatility shocks take time to work their way through the system (the "decay rate" of volatility) and often demonstrate persistence over several periods. Returns for some assets may be "sticky." A bond's value, for example, may fluctuate with interest rate movements that are not random but which move slowly from their current values. It is highly unlikely to see an interest rate rapidly jump from, say, 10 percent to 1 percent in a single month. One way of determining the degree to which current returns reflect past return values is to regress price change in time 't' against price change for time 't-1' (an autocorrelation process of order one), time 't-2' (an autocorrelation process of order two), and so on. Over time, positive autocorrelation may make the return series more risky than a strict Gaussian return series distribution. A negative return in period one is more likely to be followed by a negative return in period two, and so forth. While persistence in positive returns is welcome; persistence in negative returns can be catastrophic to wealth. If the Sharpe Ratio is used to measure the reward to risk tradeoff in an autocorrelated time series, it may produce spurious results without appropriate adjustment.[18]

In addition to autocorrelation, a time series distribution can exhibit a shape that differs considerably from that of a symmetric bell curve. Of special interest are the statistical values of the skew and kurtosis parameters. Distributions evidencing a long left-side tail (negative skew) are more likely to generate large losses; distributions evidencing a long right-side tail (positive skew) are valued by investors because of their propensity to produce large gains.[19] Likewise, non-normal distributions may evidence "fat tails" (leptokurtic distributions). *A fat tailed distribution is more likely to produce extreme results (both positive and negative) than a bell curve distribution. Together, negative skew and positive kurtosis can combine to produce extreme losses at a magnitude and frequency greater than that found in a normal distribution*. Again, without an appropriate adjustment to the Sharpe Ratio, an investor will not have a credible reward to risk ratio.[20]

An Example

Here is an example of Sharpe Ratio adjustments given a series of monthly investment returns over a five-year period. The returns are listed in the following table:

Month #	Monthly Return of Portfolio	US 30 Day T-Bill Return
1	-0.0223	0.0032
2	0.031	0.0031
3	-0.0383	0.0026
4	-0.0252	0.0026
5	0.007	0.0023
6	0.0404	0.0023
7	0.0127	0.0028
8	0.0182	0.0023
9	0.0341	0.0022
10	0.024	0.0025
11	0.0115	0.0024
12	0.0302	0.0022
13	0.0061	0.0025
14	0.00	0.0024
15	0.0416	0.0025
16	-0.0073	0.0026
17	0.0076	0.0022

18	-0.0108	0.0025
19	-0.0156	0.0023
20	0.0569	0.0025
21	-0.0224	0.0021
22	-0.0386	0.0027
23	0.0136	0.0027
24	0.0181	0.0032
25	-0.0245	0.0031
26	0.0368	0.0028
27	0.047	0.0037
28	-0.0288	0.0037
29	0.024	0.0038
30	-0.0382	0.0037
31	0.0128	0.0044
32	0.0184	0.0042
33	0.0433	0.0040
34	0.0323	0.0046
35	0.0298	0.0044
36	-0.0429	0.0054
37	0.0107	0.0047
38	0.0401	0.0045
39	0.0149	0.0047
40	0.023	0.0043
41	-0.0305	0.0047
42	0.0516	0.0042
43	0.0177	0.0049
44	0.0243	0.0043
45	0.0147	0.0039
46	0.0227	0.0039
47	0.0331	0.0046
48	-0.0124	0.0042
49	-0.0127	0.004
50	-0.0419	0.0045
51	-0.1357	0.0041
52	-0.0312	0.0044
53	-0.0161	0.0042
54	0.0863	0.0041
55	-0.0137	0.0046
56	0.0366	0.0045
57	0.0074	0.0039
58	-0.0188	0.0043
59	0.0383	0.0043
60	0.066	0.0049

The average annual return of the portfolio is 9.14 percent with an annual standard deviation of 12.09 percent. The Sharpe Ratio, assuming a normal distribution, is 0.4030. The adjustment for autocorrelation, however, reduces the ratio to 0.3726; and, the further adjustment for skew and kurtosis reduces the ratio value to 0.3406. The combined adjustment results in a 15 percent reduction in the value of the Sharpe Ratio. Given the statistical characteristics of *the empirical distribution*, the reward to risk tradeoff is significantly less attractive than that suggested by assuming parameter values for a *normal distribution*. The next sections of this article show how even these adjustments can fail to reveal substantial hidden risks.

Not surprisingly, one of the most pernicious abuses of the Sharpe Ratio is the calculation of the ratio's value assuming a normal distribution when, in fact, the return distribution is patently non-normal. Some Hedge Fund promoters have been accused of using the Sharpe Ratio to compare Hedge Fund portfolios with non-linear payout functions to the return series of financial assets like the S&P 500 stock index. This is truly an apples-to-oranges comparison. The text-book example of such a fraud is the Hedge Fund manager who elects to write deep, out-of-the-money put options on a highly leveraged basis. All fund returns will be positive until the day when the puts are suddenly in-the-money and fund investors are wiped out. Until that day, the fund's Sharpe Ratio will be unbeatable!

UNCOVERING HIDDEN RISK: DISTRIBUTIONAL UNCERTAINTIES

The empirical distribution of financial asset returns represents a single sample of historically realized investment results. Making risk assessments based on a single sample gives rise to at least two statistical issues: (1) if the sample is not representative, then any parameter estimates based on it will not be accurate, and (2) if the sample is not produced by a single underlying return generation process, then any statistical measurements based on *the central limit theorem* are unreliable. The first issue should make the trustee distrustful of any prediction that assumes the future will be exactly like the past.[21] The second should make the trustee distrustful of any prediction that assumes that distributional parameters like mean return and variance are constant rather than time varying.

The central limit theorem allows us to be comfortable with history. The longer the sequence of historical returns (i.e., the more data), the more accurately we can describe portfolio behaviors. Although there is only a finite sample of returns, nevertheless, as the sample grows larger, the values of the key statistical parameters that characterize the return distribution will, under the central limit theorem, converge to their true values. Stated otherwise, there is a long-term average expected return which represents the central tendency for the growth of wealth. Of course, the expected long-term return differs according to the investor's asset allocation. In general, an allocation heavily weighted towards stocks has a higher long-term expected return compared to an allocation heavily weighted towards bonds. In any period, realized returns may be either above or below this central tendency; but, such deviations represent only temporary diversions from the "true" central mean (called "the first moment").[22] Likewise, depending on the method of measurement, by the central limit theorem, there is a constant long-term parameter value for volatility (called "the second moment").

Risk-averse investors have a positive preference for a high first moment (average return) and a negative preference for a high second moment (volatility)—they like return and dislike risk. Although this economic world view allows for period-by-period variations in realized risks and returns, such variations are merely temporary perturbations in fixed long-term parameter values. Volatility differs from period-to-period; but its long-term parameter value is a constant—i.e., not time varying. Correlation—as the byproduct of asset returns and volatility—is also deemed to converge to an average or theoretical steady-state value. Econometricians call this constant value *unconditional correlation*. Under the central limit theorem, the larger the sample (i.e., the longer the history of returns), the more confident the investor can be in the "true" (i.e., unconditional) value of asset correlations as well as the value of distributional parameters. These assumptions provide a strong theoretical justification for a "stay-the-course" asset management strategy. Parameter values may hop around but they revert to the mean.

Beginning in the late 1980s more powerful computers allowed financial economists to model asset returns in such a manner that volatility in risk models became volatile (time varying volatility) and correlations became dynamic (conditional correlation v. unconditional correlation). Some current asset pricing theories now view mean and volatility not as parameters that necessarily converge towards a theoretical constant value, but, rather, as dynamic values that must be adjusted both within differing regimes and across differing regimes. There may not be a true overall unconditional average like the central limit theorem suggests; rather, volatility and correlation values are conditional on the particular market regime ("bull" or "bear" market). On the other hand, there is evidence suggesting that long-term stock returns are, in fact, mean reversionary. If this is the case, stocks may be a safe asset for long-term investors. As of the current time, there is no consensus view on many aspects of this controversy. However, econometricians generally acknowledge that the distribution of financial time series is not normal.

RISK MODELS AND SIMULATION

Monte Carlo simulation is a common method of using the single path of historical returns to generate a large set of possible future price paths. The computer's random number generator produces many thousands of mathematically probable return sequences despite the fact that the sequences may never manifest themselves empirically. The ability to manufacture a rich set of future return possibilities is a great advantage of a simulation approach. However, many simulations reflect risk models that operate under the twin assumptions of (1) a normal distribution of returns, and (2) fixed parameters.

The following section explores the economic consequences of restricting the distributional assumptions underlying the risk model. The model simulates three diversified portfolio asset allocations (40 percent stock – 60 percent fixed income; 60 percent stock – 40 percent fixed income; and, 80 percent stock – 20 percent fixed income) over a 10-year planning horizon. The experiment is as follows:

1. The first test examines a risk model that uses unadjusted historical returns under the assumption that the underlying distribution is normal. This approach also takes it for granted that the realized sample of historical returns is representative of the true, but unobservable, return generating process. In other words, although we do not know the sequence of future returns, we trust that their distribution will have parameters similar to the distribution of past returns.

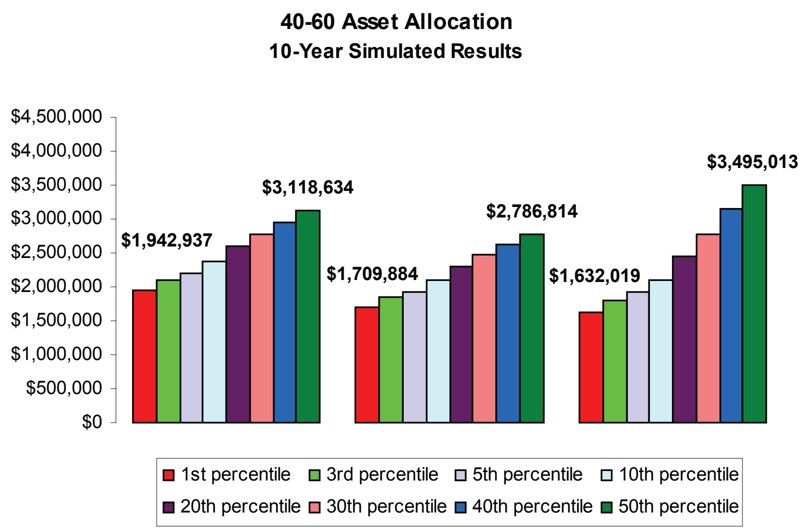
2. The second risk model uses the Sharpe Ratio to adjust historical returns to conform to the Capital Asset Pricing Model's assumption that, in an efficient market, all assets have the same long-term expected real Sharpe Ratio. Adjusted returns are called "efficient returns." As in the first test, the second test assumes standard linear time-invariant parameter values.
3. The third risk model does not assume a single normal distribution. Rather, it divides the sample of historical returns into two regimes—a bear market regime with parameter values reflective of poor economic states; and a bull market regime with parameter values reflective of prosperous economic states.

The goal is to determine the trustworthiness of risk measures like the Sharpe Ratio when their calculated values are based on simplified assumptions concerning the nature of the return generating process.

The first test simulates a return distribution that might be termed a "factor model distribution." For example, if the historical returns reflect a greater-than-expected reward for investing in small and value stocks given their historical standard deviations, then the simulation model will continue to draw from a distribution characterized by small and value premia. The second test moves towards an "equilibrium model distribution." The adjustment to returns is in the spirit of the single-factor Capital Asset Pricing Model in that it assumes a linear and strictly proportional relationship between risk, as measured by standard deviation, and return. In both the factor model and the efficient returns model, the conditional means and variances remain time invariant. The third test simulates time varying parameters in which both volatility and expected return differ according to whether the economy is in a recession or boom state. Most time series studies indicate both an economically and statistically significant difference in conditional means and volatilities across these two states. In the recessionary state, results can easily turn uglier than expected. A single normal distribution tends to average-out downside risks. This is because of the assumption that they are outliers from a mean calculated by aggregating returns across all economies—both good and bad. However, a regime switching model using conditional parameter values is not subject to this type of misspecification.[23] A time series analysis based on normality assumptions often has difficulty "explaining" both sudden and severe downside price moves from events like the October 1987 market crash, and persistent depression of prices evidenced by the failure of the Japanese stock market to recover fully from its fall in the early 1990s. A Monte Carlo simulation based on a regime-switching model avoids many of these traps.[24] Thus, the third test moves away from equilibrium and simple linear models. The Appendix following provides details on data series and other model inputs. The initial portfolio value is $2 million. All values are expressed in constant dollars.[25]

Graph three depicts terminal wealth results at selected percentiles of the cumulative distribution for a 40 percent stock/60 percent bond allocation. A bookkeeping algorithm ranks simulation results from worst to best according to ending portfolio dollar values. Percentile one ("worst case") is the dollar value of the 50th trial (5,000 x .01 = 50). Specific percentile-one values appear above the far left-hand column. Median (50th percentile) values appear above the far right-hand column. The graph does not track above-median results.

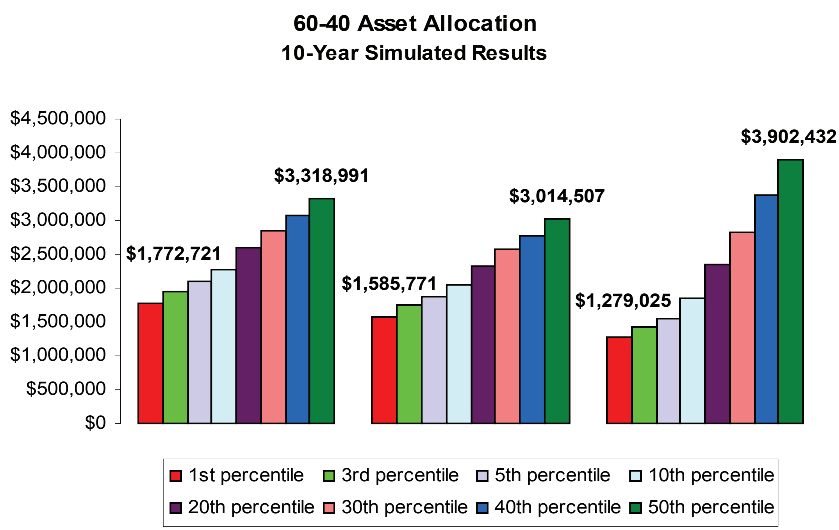
Graph Three



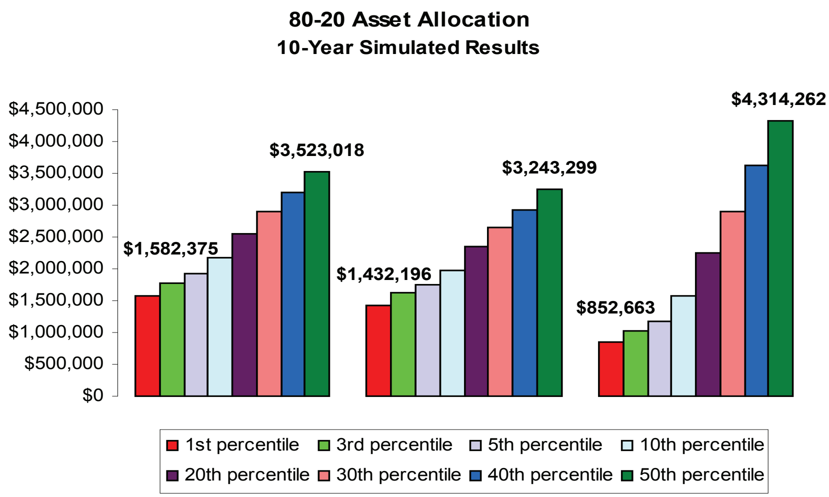
It is apparent that simulated outputs differ significantly. Monte Carlo simulations based on a normal distribution which inputs unadjusted historical returns fails to capture important properties of time series data. Inputting returns based on a CAPM real Sharpe Ratio adjustment changes the values of the distribution and indicates a greater downside risk to the 40/60 allocation. It is, of course, possible to use a student-*t* distribution to capture the "fat-tail" downside risk phenomenon.[26] However, this still begs the question of how time varying volatility and correlation relationships affect downside risk probability. Models that break away from single, fixed-parameter Gaussian distributions are better able to capture important time series behaviors. As detailed in the Appendix, a regime-switching (Markov transition model) is well suited to model the reward to-risk-tradeoffs faced by trustees.

Graphs four and five further indicate the dangers of using misspecified simulation models. As the portfolio increases equity weighting, the differential in downside risk magnifies. Using a single Gaussian distribution with unconditional parameter inputs distorts the real risk of the investment venture. Econometric studies indicate that empirical downside volatility is more persistent than implied in a single value parameter model. This means that downside shocks may persist at a frequency and magnitude greater than suggested by Monte Carlo simulations based on a normality assumption.

Graph Four



Graph Five



The divergence in the models' downside risk measurement is truly breathtaking. An investment allocation to 80 percent stocks may seem prudent and suitable for an investor with a long-term planning horizon when return probabilities are modeled under the assumption of normality. Such an allocation, however, may be unacceptable when risk is modeled more realistically. The patterns of financial asset behaviors that are critical for correct specification of the reward-to-risk tradeoffs are best captured by models that employ conditional parameter values reflective of multiple economic states.

How do these results relate to the Sharpe Ratio? Initially, we argued that the Sharpe Ratio is a tool that is best used when the return distribution can be fully characterized by its mean and variance parameters. The family of symmetric distributions is commonly modeled by the normal distribution; and, the Sharpe Ratio yields a precise measure of reward and risk only when returns are both independent (like coin flips) and identically distributed (symmetrical). Although selecting a normal or log-normal distribution has certain advantages, the Sharpe Ratio yields imprecise reward/risk measures unless the historical returns are adjusted for autocorrelation and excess kurtosis. If returns are adjusted towards a general single-index equilibrium model ("efficient returns"), it appears that downside risk increases relative to a multi-factor model. Although such a conclusion may be controversial (i.e., factor exposures may in fact represent priced risks which are captured by higher distributional moments such as negative skew), nevertheless, the Sharpe Ratio is happier living in this type of modeling environment. If factor loadings (e.g., value and small) do not generate expected rewards, the long-term investor has a better picture of downside risk under the efficient return model. Conversely, if factor loadings pay off, an investor who elected to avoid them may still make some attractive returns. The Appendix provides information on the full reward/risk tradeoff over one-, five-, and ten-year periods.

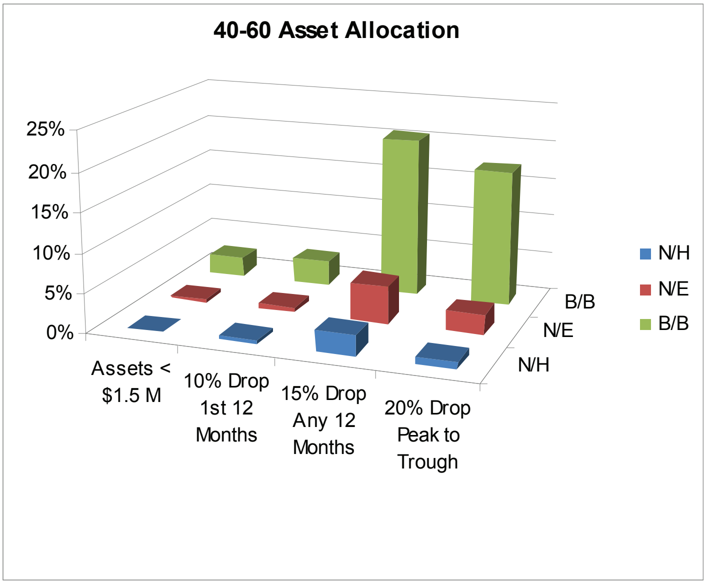
If the Sharpe Ratio can, with certain algebraic adjustments, live within the first two simulation models, it finds the environment of the third model very uncomfortable. We can no longer speak of a single unconditional Sharpe Ratio. Rather, the Ratio values must be conditioned upon the economic regime (Bull or Bear). Thus, the conditional Sharpe Ratio in a Bear market regime may be negative; and, as discussed, may present interpretive difficulties.

The next series of graphs expand the view of downside risk to encompass the probability of the following events:

1. The value of the $2 million portfolio drops, at any time over the ten-year planning horizon, to an inflation-adjusted value of $1.5 million or less;
2. The portfolio's value drops by 10 percent or more in the initial 12 months following portfolio implementation;
3. The portfolio's value drops by 15 percent or more in any 12 month period following portfolio implementation; and,
4. The portfolio's value drops by at least 20 percent peak to trough in any time period.

These risk measures provide greater insight into (1) the importance of modeling risk with time-varying parameters, and (2) the limitations of a constant-parameter reward-to-risk measure such as the Sharpe Ratio.

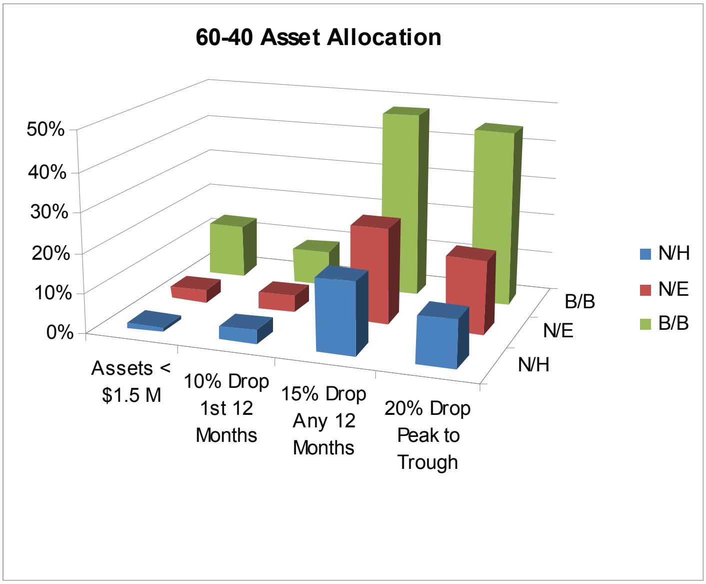
Graph Six



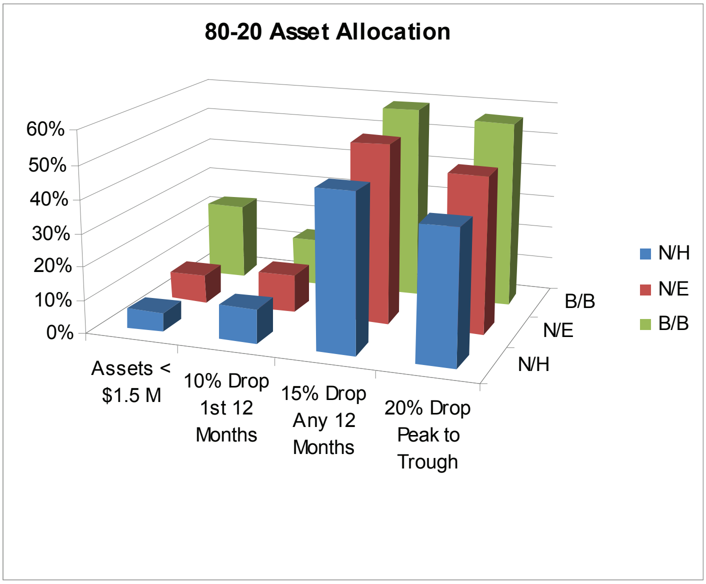
The first row (designated N/H) represents a Normal distribution using unadjusted historical returns; the second row (designated N/E) represents a Normal distribution using efficient returns; and the back row (designated B/B) represents a Bull/Bear regime switching model using conditional parameter values.

Across all measures of downside risk, it is clear that investors are best served by simulation models that emulate important financial time series characteristics. It is no surprise that, as the investor increases the commitment to equity, the probability of encountering substantial downside risk increases. What is noteworthy, however, is that the probability of adverse results is considerably higher than predicted when risk is modeled under the assumption of distributional normality. If investment prudence requires "… an overall investment strategy, which should incorporate risk and return objectives…,"[27] it is critical that trustees use a good risk model.

Graph Seven



Graph Eight



Trustees often use the Sharpe Ratio as a performance ranking metric. However, the asset allocation which historically has made the most efficient use of risk may not be the allocation best suited to the investor's goals and circumstances. Almost certainly, the Sharpe Ratio cannot yield a precise measure of the reward-to-risk tradeoff faced by investors owning financial assets. Rewards and risks are asymmetric and, as economic theory suggests, are related to conditions in the real economy. Part Two outlines how trustees can develop portfolio preferencing criteria that overcome the limitations of MPT ratios.

APPENDIX: SIMULATION DATA AND METHODOLOGY

The Simulation Model

Simulation is an approach to modeling that seeks to mimic a functioning system as it evolves. It is built on mathematical equations that express the assumed form of the system's operation. Simulation models assume a range of complexity from (1) a simple "bootstrap" of time series data in which periodic returns are sampled with replacement to create a large number of re-shuffled return sequences; to, (2) a simple structural model like Monte Carlo simulation which draws random samples from a pre-specified distribution; to (3) more complex simulation models that blend various types of distributions or alternate distributions according to certain probability criteria. Whenever portfolios operate under conditions of cash flows (dollars going into or out of the portfolio), simulation analysis is an indispensible tool for evaluating the likelihood of economic success or failure.

The simulation model used in this paper incorporates several "moving parts" which are best characterized as random variables. These include:

The Planning Horizon—the applicable planning horizon can either be fixed (e.g., a university endowment will need to finance a new building in exactly seven years) or variable (an indeterminate length). When the planning horizon is measured by life span, the application simulates sample lifetimes using a Society of Actuaries annuity table based on "white collar" retirees from Defined Benefit Pension Plans. This table is conservative (i.e., exhibits a force of mortality lower than general population tables used by Social Security); and, therefore suggests a higher likelihood of a long life. Unless otherwise indicated, the simulation reflects longevity expectations that assume good health.

The Economy—the simulation application divides economies into two regimes: A Bear Market regime (defined as a 20 percent or greater peak to trough price decline for the Capital Appreciation S&P 500 stock index); and, a Bull Market regime. Using historical data from January 1973 through the end of last year, the historical lengths of Bull and Bear Markets are determined. The simulation uses a Markov-switching regime model (with a random selection for the initial economic regime) to determine the sequence of market conditions that will be faced by the investor. The probability (p) that the initial economy is in a Bull Market regime or a Bear Market regime (1 – p) is based on historical frequencies. For all future periods, the simulation determines the probability of remaining in a Bear Market given that the last month was a Bear; or, of switching from a Bear to a Bull Market given the total duration of the Bear Market to date. Similar calculations are made for the probability of remaining in or leaving a Bull Market regime.

Inflation—the model proxies inflation by the Consumer Price Index. Changes in the inflation rate are primary determinates of the likelihood that periodic investment returns are either positive or negative. The econometric model specifies the inflation generating process as a serially correlated random variable with a "smoothed" reversionary factor. Specifically, the algorithm regresses the average value of the previous 12 month's inflation against the average value of the next 12 months. The value is calculated as:

Inflation_t = long-term inflation + Persistence Coefficient [Sum(inflation_{t-1} … inflation_{t-12}) / 12 – long-term inflation] + error term

Where the error term is an iid, "white noise" process.

When the application has not yet produced 12 monthly simulated values, the application recursively calculates the average of the preceding 12 months by using the initial value to replace any missing terms. Therefore, the value for average prior 12 month inflation in the second month is 11/12 * the initial value + 1/12 * the value in the first month.

The persistence Coefficient determines the speed of CPI mean reversion. The Coefficient's value is calculated via a regression of the rolling 12-month CPI against the rolling forward 12-month CPI. Thus the model assumes that inflation is an Ornstein-Uhlenbeck process which includes a term for autocorrelation as well as for mean-reversion.

Investment Returns—the simulation model generates investment returns utilizing common matrix algebra techniques. Utilizing separate variance/covariance matrices from historical Bull and Bear market regimes, the application executes a Cholesky decomposition. It may also adjust dependence relationships by shrinking extreme off-diagonal elements to assure matrix invertibility. The Cholesky matrix algebra operation "divides" a variance/covariance matrix into upper and lower triangle matrices which make them equivalent to the square root of a variance matrix. If there exists a lower triangle matrix C such that the historical matrix $V = CC^t$, then C is a Cholesky matrix. The application simulates combinations of return series $(\vec{x})$ where each historical return series is transformed (by subtracting the mean and dividing by the standard deviation) into an independent standard normal variable $(\vec{z})$. The computer's random number generation function can readily simulate future evolutions for each independent return vector by drawing values for uncorrelated zero-mean variables. Pre-multiplying the vectors of simulated independent returns by C $C(\vec{z})$ restores their equivalency to each original return series $(\vec{x}) = C(\vec{z})$. The variance of the independent vectors is easily determined; and, pre and post multiplication of the variance of $(\vec{z})$ by the appropriate lower triangle decomposition matrix C and its inverse restores the correlation structure by generating the required variance/covariance matrix. $(V = CV(\vec{z})\, C^t)$

Financial asset return series usually cannot be characterized as normal (bell-curve) distributions. Portfolio investment risk defined by the first two moments of multivariate symmetric distributions (Gaussian, Student's *t*, Cauchy, etc.) is often misleading. Monte Carlo simulations based on a normal distribution cannot realistically capture the frequency and magnitude of tail-risk events (leptokurtosis). To avoid this deficiency, the application utilizes two normal distributions (Bull and Bear) with separately calculated means and variances for each regime. The distributions, according to the Markov transition probabilities described above, enable the model to capture the risk of outlier results that mirror real world frequencies rather than risks that are largely predetermined by theoretical parameter inputs.

Additionally, a regime switching approach captures dynamic correlation and time-varying risk premia over different market conditions. Thus, instead of using average unconditional correlation values determined by the historical data, the application applies the historical correlation values conditioned on Bull and Bear Market data. For example, over the entire sample period, an asset class may exhibit a mean of 10 percent and a standard deviation of 20 percent. However, during Bull Markets, the parameter values may be +18 percent mean and 15 percent standard deviation; while, during Bear Markets, the parameter values by be -23 percent and 25 percent respectively. Thus, simply using the unconditional mean, standard deviation and correlation values for the aggregate historical period cannot capture realistic asset price behavior.

The simulation model inputs the above random variables to drive portfolio evolutions over a wide range of possible future economies. Given the large number of simulation paths (5,000 trials), there is a rich set of future asset returns. It should be recognized, however, that any model is an imperfect representation of a more complex reality. In this case, there are (at least) two "model risks" which should be considered:

1. Investors are interested in forecasts of a price change process. However, the time series of asset prices is not statistically "stationary" (i.e., exhibits the potential for infinite variance). It is only by "differentiating" the logarithm of prices on a period-by-period basis that the creation of a stationary series of returns is possible. That is to say, it is only possible to model asset returns; but investors measure wealth based on asset prices. This is a subtle but important distinction. Returns are based on the single historical path of price changes, the realization of which is merely a manifestation of an unknown price generating process. Simulation analysis greatly broadens our perspective about possible future outcomes; but any model of such a process must remain only a crude approximation of reality. Indeed, calculation of investment return is a function of the measurement interval (yearly, monthly, daily, intraday, continuous time) and, at the limit, may be meaningless in a statistical context.
2. The single historically realized return path for each asset class may be "representative" of the unknown price generating process; or, may merely be an outlier result unlikely to persist. For example, an asset allocation tilt towards small and value stocks is based on historical return data. If the premium for investing in small and value stocks reflects a reward for systematic risks, then investors have some confidence that they will continue to be rewarded for making these investments. If, however, the

premium for such investments is merely an artifact of a chance historical price process, then investors may be increasing risk as they move their asset allocation deeper into the small/value gradient. Furthermore, investment volatility is measured by the variance statistic [the squared difference between actual returns and average return]. But if the historical return path is not representative, then the concept of average becomes meaningless and statistical measures are not illuminating.

Investors are rewarded for taking prudent and calculated risks. Investors may use the historical data to make inferences concerning the interrelationships between asset allocation, risk and reward. However, in designing and implementing a portfolio, it is always wise to remain aware of parameter uncertainties. Past performance is not a guarantee of future results. A primary purpose of this study is to determine the affects of certain parameter changes with respect to the risk/reward tradeoffs faced by trustees.

Inputs for Accumulation Risk Model Simulation Study

The model assumes a $2 million initial portfolio value. There are three macro-allocations: 40 percent stock/60 percent fixed income, 60 percent stock/40 percent fixed income, and 80 percent stock/20 percent fixed income as follows: [28]

Asset Class	Proxy Benchmark Index	40/60	60/40	80/20
U.S. Large Cap Stocks	S&P 500	8%	12%	16%
U.S. Large Cap Value Stocks	Fama/French US Large Value	7%	10.5%	14%
U.S. Small Cap Stocks	CRSP 6-7-8 Deciles	2%	3%	4%
U.S. Small Cap Value Stocks	Fama/French US Small Value	2%	3%	4%
U.S. Securitized Real Estate	FTSE NAREIT-Equity	2%	3%	4%
Foreign Large Cap Stocks	MSCI EAFE	7%	10.5%	14%
Foreign Large Cap Value Stocks	MSCI EAFE Value	6%	9%	12%
Foreign Small Cap Stocks	DFA Int'l Small Cap	2%	3%	4%
Foreign Small Cap Value Stocks	Citigroup EMI Val EPAC	2%	3%	4%
Emerging Markets Stocks	S&P/IFCI Emerging Composite	2%	3%	4%
Short-Term U.S. Bonds	U.S. 1 Yr. Treasury Const. Mat.	15%	10%	5%
Intermediate-Term U.S. Bonds	Barclays IT Gvt/Credit	15%	10%	5%
U.S. Inflation-Protected Treasuries	Barclays Global Real U.S. TIPS	15%	10%	5%
Global Bonds	Citigroup World 1+ Yr. Govt.	15%	10%	5%

The simulation generates 5000 trials each of which has a fixed length of 120 months. It assumes a buy-and-hold asset management approach with no fees or transaction expenses. All outputs are adjusted for CPI (i.e., expressed in constant dollars). No distributions are taken from the portfolio during the planning horizon. The input data consists of the time series of monthly returns over the period 1973 through 2008.

If the benchmark index did not exist for the entire period, the covariance matrix is calculated based on partial data with shrinkage of off-diagonal elements if necessary to insure invertibility.

The effects of three types of distributional assumptions on the cumulative probability function of wealth over periods of one, five and 10 years were tested:

1. A Normal Distribution model which inputs unadjusted historical returns;
2. A Normal Distribution model which inputs "efficient" returns. Efficient returns are risk-adjusted on a linear basis in the spirit of the Capital Asset Pricing Model. The adjustment creates equal, real, long-term volatility-adjusted values for all asset classes. For example, if an asset class has twice the volatility of the S&P it will have twice the volatility-adjusted long-term real return. The efficiency assumption is that all assets should have the same long-term expected real Sharpe Ratio. The adjustment is as follows:
   - The expected long-term real rate of return {LTRRR} = [avg. annual return – ½ variance – CPI].
   - The expected real Sharpe Ratio of the S&P 500 = [$LTRRR_{S\&P}$ ÷ Standard $Deviation_{S\&P}$].
   - Generalizing from the S&P, the equilibrium return of other assets is calculated as:

     $$LTRRR_{asset} = \text{Real Sharpe}_{S\&P} * \text{Std Dev}_{asset} + \tfrac{1}{2}\,\text{Variance}_{asset} + CPI.$$
3. A Regime Switching Model which assumes a Bull Market regime and a Bear Market regime as explained above.

The following tables detail percentile dollar values over the cumulative probability function:

Summary of Accumulation Simulation

1-Year Results: Normal Historical Returns

Percentile	40-60	60-40	80-20
1st percentile	$1,805,373	$1,721,700	$1,625,843
3rd percentile	$1,853,862	$1,785,575	$1,712,281
5th percentile	$1,883,789	$1,822,887	$1,753,770
10th percentile	$1,921,944	$1,877,807	$1,826,593
20th percentile	$1,977,221	$1,950,565	$1,922,027
30th percentile	$2,019,588	$2,008,074	$1,996,648
40th percentile	$2,057,543	$2,058,875	$2,059,967
50th percentile	$2,090,680	$2,104,492	$2,117,202
60th percentile	$2,125,387	$2,150,417	$2,179,135
70th percentile	$2,164,077	$2,208,599	$2,252,687
80th percentile	$2,211,873	$2,272,766	$2,341,110
90th percentile	$2,278,186	$2,366,347	$2,458,479
95th percentile	$2,335,970	$2,447,031	$2,563,550
97th percentile	$2,374,973	$2,501,072	$2,631,683
99th percentile	$2,443,604	$2,609,268	$2,779,363

5-Year Results: Normal Historical Returns

Percentile	40-60	60-40	80-20
1st percentile	$1,803,350	$1,652,156	$1,471,504
3rd percentile	$1,894,479	$1,776,332	$1,645,958
5th percentile	$1,959,194	$1,853,162	$1,729,368
10th percentile	$2,058,382	$1,981,347	$1,896,148
20th percentile	$2,194,860	$2,158,777	$2,117,934
30th percentile	$2,297,815	$2,299,328	$2,298,598
40th percentile	$2,392,868	$2,435,588	$2,475,498
50th percentile	$2,490,729	$2,566,084	$2,649,641
60th percentile	$2,591,581	$2,711,994	$2,829,790
70th percentile	$2,704,177	$2,876,634	$3,052,462
80th percentile	$2,855,889	$3,076,294	$3,312,642
90th percentile	$3,082,954	$3,406,709	$3,752,472
95th percentile	$3,280,607	$3,699,982	$4,154,851
97th percentile	$3,464,837	$3,961,827	$4,449,705
99th percentile	$3,794,505	$4,444,511	$5,102,592

10-Year Results: Normal Historical Returns

Percentile	40-60	60-40	80-20
1st percentile	$1,942,937	$1,772,721	$1,582,375
3rd percentile	$2,104,953	$1,946,981	$1,778,075
5th percentile	$2,189,610	$2,087,784	$1,934,050
10th percentile	$2,362,786	$2,276,932	$2,177,073
20th percentile	$2,588,512	$2,588,175	$2,557,622
30th percentile	$2,770,948	$2,839,901	$2,893,957
40th percentile	$2,941,205	$3,066,041	$3,188,661
50th percentile	$3,118,634	$3,318,991	$3,523,018
60th percentile	$3,302,009	$3,575,609	$3,862,759
70th percentile	$3,534,781	$3,911,788	$4,313,990
80th percentile	$3,844,854	$4,354,606	$4,873,399
90th percentile	$4,349,239	$5,109,631	$5,873,156
95th percentile	$4,842,612	$5,853,819	$6,882,702
97th percentile	$5,258,922	$6,458,158	$7,641,818
99th percentile	$6,130,279	$7,824,756	$9,529,262

1-Year Results: Normal Efficient Returns

Percentile	40-60	60-40	80-20
1st percentile	$1,783,336	$1,704,210	$1,612,876
3rd percentile	$1,832,577	$1,767,204	$1,697,753
5th percentile	$1,861,648	$1,804,580	$1,740,155
10th percentile	$1,899,840	$1,859,396	$1,812,078
20th percentile	$1,954,456	$1,931,418	$1,907,670
30th percentile	$1,996,536	$1,988,655	$1,981,502
40th percentile	$2,034,547	$2,039,412	$2,044,173
50th percentile	$2,067,485	$2,084,873	$2,101,052
60th percentile	$2,101,704	$2,130,871	$2,162,509
70th percentile	$2,140,256	$2,188,285	$2,236,055
80th percentile	$2,187,477	$2,251,935	$2,323,550
90th percentile	$2,253,970	$2,345,637	$2,441,327
95th percentile	$2,310,479	$2,424,963	$2,545,668
97th percentile	$2,349,798	$2,479,368	$2,613,358
99th percentile	$2,418,080	$2,589,191	$2,760,487

5-Year Results: Normal Efficient Returns

Percentile	40-60	60-40	80-20
1st percentile	$1,696,177	$1,564,278	$1,406,397
3rd percentile	$1,782,867	$1,682,832	$1,570,500
5th percentile	$1,842,826	$1,757,287	$1,654,445
10th percentile	$1,938,253	$1,884,113	$1,817,558
20th percentile	$2,070,458	$2,053,741	$2,032,088
30th percentile	$2,166,903	$2,190,967	$2,207,290
40th percentile	$2,259,105	$2,321,547	$2,376,166
50th percentile	$2,353,798	$2,449,096	$2,549,297
60th percentile	$2,450,982	$2,589,316	$2,724,361
70th percentile	$2,562,027	$2,750,105	$2,937,748
80th percentile	$2,708,499	$2,943,220	$3,195,216
90th percentile	$2,926,568	$3,263,470	$3,621,766
95th percentile	$3,116,746	$3,549,256	$4,011,100
97th percentile	$3,292,989	$3,806,973	$4,301,593
99th percentile	$3,621,256	$4,270,302	$4,927,848

10-Year Results: Normal Efficient Returns

Percentile	40-60	60-40	80-20
1st percentile	$1,709,884	$1,585,771	$1,432,196
3rd percentile	$1,852,835	$1,744,127	$1,612,905
5th percentile	$1,935,987	$1,867,450	$1,759,973
10th percentile	$2,094,012	$2,040,261	$1,986,514
20th percentile	$2,300,343	$2,326,716	$2,348,015
30th percentile	$2,467,411	$2,568,068	$2,661,477
40th percentile	$2,622,710	$2,776,874	$2,935,733
50th percentile	$2,786,814	$3,014,507	$3,243,299
60th percentile	$2,952,904	$3,261,071	$3,558,468
70th percentile	$3,173,040	$3,578,611	$3,979,218
80th percentile	$3,452,746	$3,974,953	$4,508,784
90th percentile	$3,942,687	$4,684,146	$5,430,669
95th percentile	$4,398,288	$5,380,075	$6,342,295
97th percentile	$4,804,114	$5,955,746	$7,110,638
99th percentile	$5,633,612	$7,227,043	$8,830,265

1-Year Results: Bull/Bear Regime Switch

Percentile	40-60	60-40	80-20
1st percentile	$1,690,555	$1,541,798	$1,373,244
3rd percentile	$1,769,744	$1,638,385	$1,513,231
5th percentile	$1,813,079	$1,699,084	$1,583,592
10th percentile	$1,890,791	$1,809,625	$1,723,540
20th percentile	$1,981,462	$1,958,150	$1,934,482
30th percentile	$2,039,621	$2,040,654	$2,039,882
40th percentile	$2,084,265	$2,103,111	$2,121,238
50th percentile	$2,121,405	$2,156,891	$2,191,066
60th percentile	$2,156,657	$2,205,610	$2,256,664
70th percentile	$2,195,879	$2,257,734	$2,323,164
80th percentile	$2,242,183	$2,325,068	$2,411,228
90th percentile	$2,309,176	$2,418,283	$2,525,759
95th percentile	$2,362,274	$2,483,664	$2,612,616
97th percentile	$2,391,586	$2,533,996	$2,676,848
99th percentile	$2,458,439	$2,618,803	$2,799,335

5-Year Results: Bull/Bear Regime Switch

Percentile	40-60	60-40	80-20
1st percentile	$1,499,552	$1,199,154	$854,890
3rd percentile	$1,655,507	$1,355,459	$1,040,950
5th percentile	$1,737,941	$1,473,622	$1,193,273
10th percentile	$1,905,383	$1,698,091	$1,496,557
20th percentile	$2,137,087	$2,049,833	$1,974,880
30th percentile	$2,353,763	$2,383,461	$2,417,196
40th percentile	$2,512,699	$2,638,343	$2,749,846
50th percentile	$2,666,942	$2,848,461	$3,033,546
60th percentile	$2,814,599	$3,068,258	$3,324,639
70th percentile	$2,994,195	$3,323,529	$3,652,702
80th percentile	$3,192,411	$3,613,610	$4,035,518
90th percentile	$3,479,119	$4,029,729	$4,594,848
95th percentile	$3,729,157	$4,385,026	$5,046,746
97th percentile	$3,881,182	$4,632,905	$5,393,242
99th percentile	$4,255,462	$5,140,946	$6,082,863

10-Year Results: Normal Efficient Returns

Percentile	40-60	60-40	80-20
1st percentile	$1,632,019	$1,279,025	$852,663
3rd percentile	$1,809,489	$1,435,935	$1,034,301
5th percentile	$1,923,219	$1,560,871	$1,183,479
10th percentile	$2,109,577	$1,846,746	$1,568,955
20th percentile	$2,459,015	$2,340,392	$2,237,803
30th percentile	$2,779,163	$2,824,317	$2,890,736
40th percentile	$3,141,948	$3,382,709	$3,622,863
50th percentile	$3,495,013	$3,902,432	$4,314,262
60th percentile	$3,902,623	$4,497,190	$5,086,574
70th percentile	$4,356,196	$5,211,405	$6,064,224
80th percentile	$4,902,482	$6,017,960	$7,120,899
90th percentile	$5,822,441	$7,334,757	$8,883,111
95th percentile	$6,599,328	$8,545,778	$10,443,680
97th percentile	$7,184,401	$9,355,862	$11,527,903
99th percentile	$8,303,673	$11,083,716	$13,751,493

NOTES

[1] *Restatement of the Law Trusts*, Volume 3, §90 [General Standard of Prudent Investment].

[2] "The trustee must give reasonably careful consideration to both the formulation and the implementation of an appropriate investment strategy, with investments to be selected and reviewed in a manner reasonably appropriate to that strategy." §90 Reporter's comment d. General requirements of care and skill.

[3] A trustee cannot substitute personal risk/return preferences and constraints for those of a trust under his administration. *See, for example*, *Buder v. Satore* (774 P.2d 1383, 1390 (Colo. 1989)) in which the Colorado Supreme Court rejected an argument by the trustee/parent that trust investments were prudent because he invested his personal capital identically to that of a trust for the benefit of his minor children.

[4] In one instance, a settlement was quickly forthcoming when Plaintiff Counsel ascertained that the trust officer's compensation bonus was linked to the outperformance of the S&P 500 stock index. This is an extreme example of the non-satiation principle: more money is always better than less. It was the imprudent pursuit of the money (nowhere in the trust instrument did the settlor implicitly or explicitly direct the trustee to beat the market) that produced the fiduciary breach complaint.

[5] *See, for example*, "The Kiss of Death: A 5-Star Morningstar Mutual Fund Rating?," Morey, Matthew R., *Journal of Investment Management* (2nd Quarter, 2005), pp. 41-52; and, "History of the Forecasters," Brooks, Robert and Gray, Brian J., *The Journal of Portfolio Management*, (Fall, 2004), pp. 113-117.

[6] For a discussion and literature review, *see* Collins, Patrick J., "'Without More': Trust Investment Manager Selection and Retention," *The Banking Law Journal* (May, 2008), pp. 391-456.

[7] Prudent investment appraisal echoes the famous legal discussion of trustee prudence advanced by Bevis Longstreth: *"In light of the overreaching principle, reaffirmed by most soundly reasoned cases and recent legislative and administrative developments, that prudence is a test of conduct and not performance, the most promising vehicle for accomplishing that shift is a paradigm of prudence based above all on process.* (emphasis added).

[8] It is interesting to note that television commercials for discount brokers currently offer investors free access to broker research reports. At one time, such reports were jealously guarded resources. One wonders if they are now priced correctly given the record of Wall Street prognostications.

[9] The perceived need for a risk-adjusted performance evaluation metric motivated

development of mathematical techniques for risk decomposition—splitting the time series of investment returns into smaller components each of which reflects a separate and uncorrelated risk exposure (e.g., principal components analysis). Such mathematical decompositions underlie many multifactor portfolio risk models currently in vogue.

10 The investor prefers to own a dollar in a "low-consumption" state such as an economic depression more than in a "high-consumption" state such as an economic boom.

11 In the late 1980s Sharpe introduced a method to evaluate active portfolio management that he named the "Sharpe Selection Ratio." This performance measure is derived from an analysis of residuals from a constrained multi-factor regression analysis and should not be confused with the Sharpe Ratio discussed in this essay.

12 The "market" is usually proxied by a broad stock index such as the MSCI World Market Index.

13 Multiplication of the right side of this equation by 1 [or, σ_m / σ_m] yields the following expression:

Expected Return = R_f + [(R_m) – R_f] x [σ_{mp} / σ_{mm}] or, Expected Return = R_f + [(R_m) – R_f] x [σ_{mp} / σ^2_m], where the last term on the right (covariance of the portfolio and the market ÷ variance of the market) is the definition of Beta. This is the classic expression of the Capital Asset Pricing Model: Expected Return = R_f + Beta[(R_m) – R_f].

14 In mathematical terms, the Ratio is "scale invariant." Linearity, of course, assumes that the investor can borrow and lend at the same rate.

15 We use the phrase "back-of-the-envelope" because the true global market is not identifiable. The slope of the return/risk tradeoff changes if the market includes non-liquid assets (privately held real estate or closely held businesses), projected labor income, other assets (baseball cards & beanie babies), or entitlements like social security retirement benefits.

16 Example from Pezier, Jacques, "Maximum Certain Equivalent Excess Returns and Equivalent Preference Criteria Part I - Theory," *ICMA Discussion Papers in Finance* DP2008-05 (December 21, 2008), pp. 6 -7.

17 Investment B manifests first-order *stochastic dominance*. The statistical concept of stochastic dominance involves comparison of cumulative distribution functions, and is an important portfolio preferencing measure.

18 The appropriate adjustment is to calculate the standard deviation of returns with the following scaling factor:

$$\sqrt{h + 2\frac{\rho}{(1-\rho)^2}[(h-1)(1-\rho) - \rho(1-\rho^{h-1})]}$$

where h is the number of returns per year and ρ is the value of the autocorrelation coefficient.

[19] It is important to acknowledge that variation in the skew parameter also changes the mean of the distribution. Strongly positive skew could mean that expected value shifts to the left at a magnitude that would not be pleasing to an investor.

[20] The appropriate adjustment for the skew and kurtosis adjusted Sharpe Ratio is $SR + (Skew/6)SR^2 – (Kurtosis/24)SR^3$, where SR = Sharpe Ratio, and the remaining terms are taken from a Taylor Series expansion of the function.

[21] There is a good reason why the Securities and Exchange Commission requires a prospectus to include a warning that "past performance is not a guarantee of future results."

[22] The "true" mean can only be estimated. It can never be calculated exactly because the set of historical returns is a finite sample realized over a limited number of economic conditions.

[23] In statistical parlance, the model better accounts for heteroscedasticity and autocorrelation.

[24] Volatility, for example, is asymmetric in that it tends to increase significantly during recessions. If a model cannot capture parameter asymmetries, then it may be unable to provide a credible measure of investment risk.

[25] Constant dollars are adjusted for inflation. This avoids the problem of "money illusion." The stochastic inflation adjustment decreases the nominal value of future dollars to reflect their diminished purchasing power.

[26] Pareto and Levy distributions as well as mixtures of distributions have also been proposed in the literature.

[27] *Restatement of the Law Trusts Third*, "The Prudent Investor Rule" §90: General Standard of Prudent Investment.

[28] Data series from Ibbotson Associates.