What Trustees Should Know about Asset Management Approaches and Rebalancing Elections

Patrick J. Collins, Ph.D., CLU, CFA
Josh Stampfli, MS (EESOR)

General Information:
Patrick J. Collins – [Contact Author] Principal: Schultz Collins Lawson Chambers, Inc., 455 Market Street, Suite 1450, San Francisco, CA 94105. Tel: 415-291-3002 / FAX 415-291-3015 [patrick@schultzcollins.com].

Josh Stampfli -- Consultant: Schultz Collins Lawson Chambers, Inc., 455 Market Street, Suite 1450, San Francisco, CA 94105. Tel: 415-291-3000

Abstract

Trust beneficiaries are best served when trustees select investment portfolio management approaches that enhance (1) the probability of achieving settlor objectives; and (2) beneficiary utility. Investors and fiduciaries would like to know, prior to implementation, the probable consequences of decisions to incur extra costs and taxes by rebalancing the portfolio to maintain investment policy guidelines and asset allocation targets. Two issues, in particular present themselves. Are there rule-of-thumb rebalance strategies that are optimal under all asset management regimes and market conditions; and, do past empirical results constitute a credible basis for making decisions regarding future rebalance elections? If terminal wealth has value because of remainder interest considerations, the trustee will select asset management and rebalance strategies to augment the utility of final dollar values. If, however, the settlor does not have these preferences, any unspent money may merely represent lost consumption opportunities for current beneficiaries. The choice of a rebalancing strategy is a function of beneficiary utility as constrained by the settlor's guidelines memorialized in the trust instrument. This essay argues that, although rebalance strategy is a critical bridge between asset allocation and trust distribution policy, there appears to be no universally optimal rule of thumb regarding either asset management approaches or the rebalance strategies designed to maintain them.

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Portfolio Management Elections: Asset Allocation & Rebalancing Strategies

A commonly used technique for achieving a well-diversified portfolio (i.e., a portfolio with risk/reward characteristics reasonably suitable to the trust) is asset allocation. Asset allocation defines an appropriate opportunity set of investments (i.e., stocks, bonds, cash) and selects the proportion or weighting of each investment within the portfolio.[1] Indeed, a portfolio may be defined as a grouping of assets; and, portfolio management may be defined as the set of tools and techniques used to make assets evolve in such a way that the trust's economic objectives are reached while respecting beneficiary needs and legitimate expectations under the settlor's constraints, preferences and guidelines.[2]

This essay focuses on one such technique—trustee asset rebalancing strategies. It argues that the future evolution of the trust's wealth may be greatly affected by the rebalance strategies employed by trustees; and, given the economic consequences of such elections, trustees should use an appropriate degree of care, skill, and caution in selecting and implementing a portfolio rebalancing strategy. Furthermore, the essay suggests that trustees should avoid *a priori* decisions regarding the optimality of a specific rebalancing strategy.[3]

It is difficult to achieve the luxury of a "rule of thumb" for portfolio rebalancing because of (1) the heterogeneity of trust instruments with respect to "…the purposes, terms, distribution requirements, and other circumstances of the trust;"[4] and, (2) the uncertainty of the underlying return generating process of risky investments. Not only might a rebalancing strategy that is optimal for one historical period be disastrous in the next; but also, a rebalance strategy that is optimal for a portfolio with one set of statistical characteristics (means, variances and correlations) may be disastrous for a portfolio with a different set. Likewise, it is perilous to apply rules derived from strategies for

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[1] Restatement Third §227 Comment 'g' Risk and the requirement of diversification: "Asset allocation decisions are a fundamental aspect of an investment strategy and a starting point in formulating a plan of diversification. They deal with the categories of investments to be included in a trust portfolio and the portions of the trust estate to be allocated to each. These decisions are subject to adjustment from time to time as changes occur in economic conditions or expectations or in the needs or investments objectives of the trust."

[2] Amenc, Noel & Le Sourd, Veronique, Portfolio Theory and Performance Analysis (Wiley Finance Series, 2003), p. 6.

[3] Restatement Third §227 Comment 'h' Prudent investment: theories and strategies: "…there are endless variations in reasonable strategies for investing and for the prudent management of risk, with a variety of legitimate theories of investment to support and incorporate into these strategies."

[4] Restatement Third §227 General Standard of Prudent Investment.

managing an asset-class / broadly diversified capital market portfolio (e.g., a portfolio of index funds) to portfolios based on more narrow sector / industry building blocks, or to portfolios based on individual tradable financial assets (stocks, bonds, futures contracts).

The selection of a suitable rebalance strategy takes shape within a legal context in that the tools and techniques suitable for a long term trust—e.g., a dynasty trust lacking specific distributional requirements, may greatly differ from those used to administer a trust required to make periodic cash distributions—e.g., a QTIP trust. The rebalance strategy's selection and implementation process also takes shape within academic and administrative contexts. Trustees, and advisors to whom they may delegate investment functions, see only the single historical path of realized returns (results), and cannot see the true but unobservable process that generates these results. By analogy, it is as if they see only a sequence of numbers, but have only the vaguest idea that rolls of one or more dice generate the numbers. They must try to fit data to models under conditions of uncertainty regarding the distribution of future returns. Under such conditions, trustees cannot be guarantors of results;[5] but, should enhance the probability of financial success by following an academically sound decision making process. Rebalancing elections are also administrative concerns in that they are one of several tools (including tax management techniques, monitoring frequency elections, performance reporting conventions, etc.) that the trustee considers. Rebalancing elections, however, are critical with respect to assuring continued alignment of the portfolio with the risk and return objectives of the trust.[6] The goal is to develop a suitable and prudent overall asset management strategy that is legally defensible, academically sound, and administratively reasonable.

Approaches to Asset Management: Fixed Mix, Tactical, and Drift

Typically, asset allocation guidelines fall into either a "fixed mix" structure[7] in which the portfolio maintains constant exposures to the risks and returns of the selected asset classes in both bull and bear market conditions; or, into a "tactical allocation" structure in which minimum and maximum levels of exposure are set for each asset class. In the latter case, the expectation is that the trustee will tilt the asset allocation weights according to market conditions, with the magnitude of the tilt conditioned upon his or her market forecasts and upon his or her confidence in the forecast's accuracy.[8] The fixed

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[5] Uniform Prudent Investor Act §8 Comment: "Trustees are not insurers. Not every investment or management decision will turn out in the light of hindsight to have been successful. Hindsight is not the relevant standard."

[6] Fabozzi, Frank J., Focardi, Sergio M. & Kolm, Petter N., Financial Modeling of the Equity Market (Wiley, 2006), p. 88: "After the asset allocation decision (a strategic decision), portfolio rebalancing (a tactical decision) is probably the second most important decision for an investment portfolio."

[7] Also known as "constant mix."

[8] Note that the fixed mix is static but "objective" while the tactical allocation is dynamic but "subjective." Some commentators believe that it is better to keep the portfolio approach "objective," and to change the asset allocation guidelines only as a response to changes in the circumstances or financial goals of the investor. The danger of basing portfolio management on "subjective" criteria is an overreaction to transitory market conditions—investing too aggressively following an up market trend (ignoring risk) and investing too conservatively following a down market trend (avoiding risk). The danger of a fixed mix strategy, however, is that it fails to take full advantage of the current information set which may contain variables with predictive content for the investor.

mix sets specific asset weighting targets while the tactical allocation structure sets ranges over which asset weightings are allowed to vary.[9]

One interesting form of tactical asset allocation that is not based on market prognostications is the "insured portfolio" approach (commonly known as "portfolio insurance").[10] Portfolio insurance sets a floor below which the investor's wealth should not penetrate. Commonly, this floor is 20% or 30% below the initial portfolio value. In addition to the floor value, the tactical allocation range sets a 'multiplier' value. When the trust's wealth is above the floor value, the multiplier is applied to the surplus to determine the portfolio's stock market exposure. For example, a $1 million portfolio may have a floor value of $800 thousand and a multiplier of 4. The surplus is ($1,000,000 - $800,000) or $200,000 x 4 = $800,000. Thus, at time zero, the portfolio is allocated $800 thousand to risky investments (stocks) and $200 thousand to risk free investments (T-Bills). If the portfolio's value moves up over time (i.e. the surplus increases) the proportion of the portfolio subject to stock market exposure also increases—a bigger cushion allows the trustee to take more risk; if the portfolio value decreases, the multiplier forces the trustee to unwind stock exposure by moving into T-Bills. Under a severe bear market condition (i.e., the portfolio loses 20% of its initial dollar value), the remaining $800 thousand is invested in T-Bills to avoid further loss. The insured portfolio approach is "active" in the sense that it responds to market conditions, but "passive" in the sense that its response is based on actual rather than predicted economic circumstances. Stock price forecasting is not a part of an insured portfolio management system.

Trustees may choose to employ active strategies to generate investment returns (focused, security selection strategies), passive strategies (broadly diversified, market-based returns provided by indexed investment vehicles) or a combination of each.[11] The question of how best to manage the aggregate portfolio is different from the question of how best to

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[9] The fixed-mix allocation is sometimes termed long-term or 'strategic' asset allocation. By contrast, tactical asset allocation is a response to market predictions for the forthcoming period: "Tactical asset allocation (TAA) … involves making short-term adjustments to asset class weights based on short-term predictions of relative performance among asset classes….TAA creates active risk….In exchange for active risk, the manager using TAA expects to earn positive active returns that sufficiently reward the investor for the risk taken." Pinto, Jerald E., & McLeavey, Dennis, W., "Strategic Asset Allocation Concepts," CFA Institute, CFA Level III Readings in Portfolio Management, (2005), p.283.

[10] Perold, Andre F., & Sharpe, William F., "Dynamic Strategies for Asset Allocation," Financial Analysts Journal (January-February, 1988), pp. 16-27. The intellectual roots of this strategy can be traced to Fischer Black and others. See, for example, Black, Fischer, "Individual Investment and Consumption under Uncertainty," Portfolio Insurance, A Guide to Dynamic Hedging, edited by Donald L. Luskin (John Wiley & Sons, 1988), pp. 207-225.

[11] Treynor, Jack L. & Black, Fischer "How to Use Security Analysis to Improve Portfolio Selection," Journal of Business (January, 1973), pp. 66-86. The Treynor/Black article describes prudent methods for blending broadly diversified indexed investments with "active views" on a subset of available securities. Depending on the level of confidence in a manager's price forecasts (parameter uncertainty regarding alpha), the trustee can mix actively managed and passive (indexed) investment management approaches in various degrees. Unlike the Treynor/Black article, this essay is neutral with respect to the debate over active vs. passive investment approaches. It concerns itself primarily with aggregate portfolio management techniques rather than with the probability that unique investment insights can yield excess alpha in a relatively efficient market for tradable financial assets.

generate investment returns. Portfolio management concerns itself with asset management policy over the applicable planning horizon; by contrast, generating period-by-period return concerns itself with security selection and market timing options.[12] Rebalancing policy is, in many ways, a bridge between the two components requiring trustees to think about how changes in investment values affect overall portfolio objectives.[13] Irrespective of the investment options selected to generate returns within each asset class, there remains the question of the management approach for the aggregate portfolio. A passive portfolio management approach is a buy-and-hold or 'drift' approach.[14] Although 'benign neglect' is not often considered prudent, this approach to trust administration, as will be demonstrated later, may be prudent for certain types of trusts.

An active portfolio management approach can take a variety of forms including tactical asset allocation (tilting the portfolio towards asset classes with higher forecasted returns); rebalancing to the fixed mix targets; or rebalancing according to the portfolio insurance multiplier's 'surplus' leverage factor. At first blush, it seems that fixed-mix rebalancing, portfolio insurance, and tactical asset allocation are first cousins because each requires the trustee (or investment manager) to take active portfolio management steps. However, in practice, tactical asset allocation is a market trend anticipating strategy. By contrast, the portfolio insurance approach to asset management is a trend following strategy.

[12] Restatement Third §227 comment 'd' General requirements of care and skill: "the trustee must give reasonably careful consideration to both the formulation and the implementation of an appropriate investment strategy, with investments to be selected and reviewed in a manner reasonably appropriate to that strategy."

[13] It should be noted that a purely "bottom up" security selection approach to generating investment returns may also approximate a passive portfolio management approach. This is because decision making focuses primarily on treasure hunting for good investments rather than on macro issues of aggregate portfolio design. Investments come and go from the portfolio not because they fulfill asset allocation or risk control functions, but merely because they are deemed to be undervalued or overvalued relative to alternatives within the investment opportunity set. Bottom up stock picking is usually considered to be the most active of investing styles because the investment manager is making a series of pure bets; and it is ironic that it entails a measure of passive portfolio drift because of the absence of guidelines. At the limit, a bottom up security selection approach transforms investment policy into a one-dimensional search for attractive investment returns. Success, in such a context, is primarily determined by future investment results. Investment results, unfortunately, are random variables; and unconstrained bottom up security selection may often be a prelude to fiduciary surcharge action. This is why, for example, the American College of Trust and Estate Counsel's "Guide for ACTEC Fellows Serving as Trustees [ACTEC Notes (Vol. 26, 2001), pp. 313-327] states: "it is generally not recommended that a stockbroker be selected as a delegated agent under the UPIA" [Uniform Prudent Investor Act]. See, also, Collins, Patrick J., "The Lawyer as Trustee: Working with Brokers, Investment Advisors & Financial Planners." Maryland Bar Journal (September, 2002).

[14] Alternately, this approach is termed a 'benign neglect' or a 'let-it-run' approach. A seminal article written by Robert Kirby in 1984 discusses a passive portfolio approach known as a "coffee can" portfolio. Initially, specific stocks are actively selected based on the investor's determination of a company's long-term prospects. However, once the selection has occurred, the stock certificates are put into a "coffee-can" which is placed out of sight for a lengthy period of time. Kirby speculates that active investment management is unlikely to add value once the portfolio is implemented. The investor either has the ability to select stocks for the long haul, or he doesn't. If such ability is present, then why incur trading costs by buying and selling during the interim periods? Kirby, Robert G. "The Coffee Can Portfolio," The Journal of Portfolio Management (Fall, 1984), pp. 76-80.

Tactical asset allocation takes a prospective view of the market; portfolio insurance takes a retrospective view of the market; and rebalancing to a fixed mix does not, in general, require a market viewpoint.

Rebalancing and Market Conditions: An Example

Even a portfolio that has all its individual investment positions passively managed (e.g., a portfolio of index funds), cannot be neglected or ignored. Irrespective of whether the investment manager takes active views regarding specific securities within the investment opportunity set, the trustee has the obligation to monitor the aggregate portfolio to assure that it continues to meet the "…trust's return requirements, risk tolerance, general purposes, specific terms, and other pertinent circumstances."[15]

There is a large body of academic literature on the topic of portfolio management. Much of the literature focuses on techniques for improving the risk/return tradeoffs faced by non-taxable investors with wealth accumulation objectives.[16] Some recent research discusses taxable investors as well as investors operating portfolios subject to periodic distributions. Research papers sometimes contrast buy-and-hold (drift) portfolio management approaches with tactical and fixed mix asset allocation oriented approaches. A survey of the extent literature reveals that research generally falls into two categories:

1. An empirical approach to the subject; or,
2. A mathematical approach to the subject.

Some papers employing an empirical (historical) approach are subject to the flaw in statistical analysis known as 'data mining.' Although the term 'data mining' covers a wide range of abuses in sound research methodology,[17] in this case it refers primarily to basing a conclusion on a sample of data that may not be representative of the true, but largely unknown, population of both past and future results. Statements such as "based on historical back testing, you should rebalance the portfolio to its asset allocation targets on a semi-annual basis in order to generate the greatest amount of ending wealth," although possibly correct in a narrowly defined sense, are merely *ad hoc* rules that either (1) do not apply to the investor (because of interim cash flows or the need for dynamic liability matching); (2) are sensitive to the beginning and ending dates of the historical period under evaluation; or (3) merely reflect past occurrences without exhibiting predictive force under future economic conditions.[18] By contrast, many useful insights

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[15] Restatement Third §227 Principles of Prudence Introduction. See also, Uniform Prudent Investor Act §2(b) Comment--Duty to Monitor: "'Managing' embraces monitoring, that is, the trustee's continuing responsibility for oversight of the suitability of investments already made as well as the trustee's decisions respecting new investments."

[16] A goal of maximizing terminal wealth subject to certain investment constraints.

[17] See, for example, Black, Fisher, "Estimating Financial Return," Financial Analysts Journal (September/October, 1993). Data mining is also termed 'data snooping.'

[18] A good example of simplistic, rule-of-thumb maxims is found in Trone, Donald B., Allbright, William R., & Taylor, Philip R., The Management of Investment Decisions, McGraw-Hill (New York, 1996) p. 215: "An optimal limit would require readjustments twice a year on average—more than twice a year and the benefits may be eroded by transaction costs…."

flow from studies using more mathematical approaches. However, the benefits of such approaches are often limited in practice by the demands imposed by intensive tax calculations, transaction costs, portfolio software limitations, limitations in investment advisor skill sets, custodian capabilities, and other real world constraints.[19]

Rebalancing is a type of active portfolio management strategy employed, depending on the study under consideration, either to enhance returns, control risk or both. A simple example illustrates both the concept and mechanics of rebalancing. Assume a portfolio that, at time zero, places $1,000 into each of three investments. This is comparable to stating that the portfolio has a one-third allocation to each asset and that the trust's investment policy requires a fixed mix asset management approach. Over the forthcoming period, investment A earns a return of 10% ($1,100); investment B earns a return of –3% ($970) and investment C earns a return of 5% ($1,050). At time period one, the aggregate portfolio has a value of $3,120. In order to maintain the one-third allocation, the investor sells $60 of investment A and $10 of investment C. The $70 in sales proceeds is reallocated to investment B. Each rebalanced investment position has a value of $1,040, which is exactly one-third of the portfolio's total period one value.

This simple example provides several interesting insights. The rebalance decision under the fixed mix asset management approach forces the trustee to sell high and buy low. This is one of the hallmarks of a contrarian market strategy.[20] Investors pursuing a fixed mix rebalancing strategy are natural counterparties to investors pursuing an insured portfolio management strategy. The fixed mix investor, in the process of portfolio rebalancing, sells insurance. Under an insured portfolio management strategy, the investor rebalances out of risky assets when they decline in value and moves into risk-free Treasuries. Fixed mix rebalancing, by contrast, demands that the trustee buy into weakness and sell into strength because money is moved out of the relative winners and into the relative losers. Thus, when seen in terms of buying or selling insurance, it seems as if rebalancing under a fixed mix approach should add value to returns (insurance has a premium cost and if you sell insurance you expect to receive a fair price for it). Paradoxically, however, the purchase of insurance mitigates risk while the sale of insurance increases risk (technically, the risk is not increased but is transferred from the buyer to the seller for consideration received). It is important to note the historical failure of the insured portfolio approach under conditions of extreme market volatility, although an extensive discussion of the reasons for its failure lie beyond the scope of this essay.[21]

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[19] Bruckenstein, Joel, "Rebalancing Act," Financial Planning Journal (May, 2006) http://www.financial-planning.com/pubs provides a critical assessment of commercial rebalancing software applications.

[20] A strict contrarian strategy, however, is a function of valuation estimation as much as it is a function of trading against the crowd. There is no forecasting in a mechanical portfolio rebalancing system. The strict contrarian investor hopes that he or she will not be "bagged" by momentum traders because of a misestimation of the intrinsic or justified value of the securities.

[21] A more comprehensive comparison of fixed-mix vs. insured portfolio management structures is found in Farrell, James L., Portfolio Management Theory and Application Second Edition (Irwin McGraw-Hill, 1997), pp. 291-297. Farrell points out that "any strategy giving a convex payoff pattern is representative of the purchase of portfolio insurance, but strategies such as the constant-mix that give concave patterns represent the sale of insurance." He concludes: "convex strategies are inherently volatile and demand significant liquidity because of the trading demanded."

In the main, reference to rebalance strategies will assume a fixed mix portfolio management approach unless otherwise noted.

Just as certain types of market conditions favor one asset allocation approach over another,[22] so also, market conditions will favor one type of rebalance strategy over another. A recent study by Vanguard, for example, details the relation between market conditions and rebalancing results.[23] The study assumes that the investor selects a fixed mix approach to portfolio management so that the investment policy's original asset allocation is maintained. The investor faces two risks: (1) relative tracking risk—the risk that the rebalance strategy will fail to keep the actual portfolio's allocation from significantly deviating from the target allocation; and, (2) absolute performance risk—the risk that the rebalanced portfolio's returns will significantly underperform those of a non-rebalanced portfolio.

The Vanguard study creates simulations of extreme and admittedly unrealistic market conditions in order to isolate the economic consequences of rebalancing decisions. In trending markets, for example, rebalancing to a target asset allocation means that assets that continue to perform strongly are sold in favor of assets that have relatively weaker performance. Although more frequent rebalancing helps the investor to control the first type of risk, it does so at the cost of increasing the second. That is to say, in a trending market, rebalancing mitigates tracking risk but generates the worst absolute return. By contrast, in mean-reverting markets rebalancing both reduces tracking risk and enhances the portfolio's returns. The risk/reward tradeoff in a mean reverting market is a function of rebalancing frequency and/or threshold bounds. It is not clear, for example, whether the increased average equity exposure that occurs as either threshold bounds widen or periodic rebalance intervals lengthen will generate returns above or below the average return improvements of rebalancing mean reverting assets. Finally, in a random-walk market environment, rebalancing also results in portfolios that more tightly track the target asset allocation (i.e., decrease tracking risk) and produce returns that only modestly deviate from those of the target asset allocation benchmark.

The simulated market conditions, however, did not incorporate factors for taxes and transaction costs. The author points out that the type of rebalancing costs (fixed costs or costs that are proportional to the amount traded) will influence the optimal rebalance strategy. If possible, rebalancing should occur with dividends, interest payments and new contributions into the portfolio. Using new money enables the portfolio manager to

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Portfolio risk control through Financial Engineering (use of derivative instruments) and Actuarial (use of annuities) approaches to asset management are important techniques that also lie beyond the scope of this essay. For a discussion of the welfare loss to owners of illiquid annuity contracts who are prevented from rebalancing to an optimal portfolio see, Browne, S., Milevsky, M.A., & Salisbury, T.S., "Asset Allocation And The Liquidity Premium For Illiquid Annuities," The Journal of Risk and Insurance (Vol. 70, 2003), No.3, pp. 509-526.

[22] Generally, an upwardly trending market favors the 'drift' and 'portfolio insurance' asset management approaches because winners are not cut back and profits accumulate unabated. Mean reverting or volatile markets favor a 'fixed mix' approach.

[23] Tokat, Yesim, "Portfolio Rebalancing in Theory and Practice," Vanguard Investment Counseling & Research (Number 31, 2006).

accomplish most of the risk-control objectives at minimum cost. The author concludes, “Just as there is no universally optimal asset allocation, there is no universally optimal rebalancing strategy. An institution selects a rebalancing strategy based on its tolerance for risk relative to a target allocation.”

Empirical Studies

Moving from a simple hypothetical three-asset rebalancing example into the study of historical data provides a variety of noteworthy and sometimes contradictory results.[24] To illustrate the empirical approach, we discuss two studies by Craig Israelsen.[25] The first, appearing in 2001, considers the historical returns of portfolios invested equally in US large company stocks, US small company stocks, and International large company stocks. Israelsen evaluates results over several time periods and compares a drifting-mix passive portfolio management approach to a constant mix (annually rebalanced) active portfolio management approach. The initial value of each portfolio is $1,000. The end-of-period results appear in the following table:

	1970-2000	1991-2000	1996-2000
Dollar Value of Drifting Mix	$42,961	$4,084	$1,803
Dollar Value of Fixed Mix	$49,496	$3,974	$1,795
Dollar benefit of annual rebalancing	$6,535	($110)	($8)
Risk reduction benefit of rebalancing as measured by % reduction in Standard Deviation of returns	18.5% less risk	23.5% less risk	17.7% less risk

These results are noteworthy because they suggest that the return enhancing benefits of portfolio rebalancing are sensitive to the time period under consideration. However, the data also suggest that annual rebalancing reduces portfolio risk. Israelsen concludes: “Asset allocation can serve the important function of dampening the downside risk without unduly penalizing return.”

His second study covers only the single twenty-five year period from 1977 through 2001. However, the study incorporates investment costs by comparing drifting-mix and fixed-mix results achieved by investors in three mutual funds (Vanguard 500 index fund, Vanguard US Small Cap index fund, and Scudder International Stock fund). The initial portfolios are each valued at $3,000 instead of $1,000; and all other assumptions are the same. In this case, rebalancing the portfolio to the fixed mix allocation generates 9% more ending wealth with 17.6% less volatility.

Unfortunately, however, the conclusions of empirical studies like Israelsen’s have only limited application. Many trust portfolios are not strictly accumulation vehicles but

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[24] An interesting example is provided in: Horvitz, Jeffrey E., “The Implications of Rebalancing the Investment Portfolio for the Taxable Investor,” The Journal of Wealth Management (Fall, 2002), p. 51: “In 1987 the stock market fell dramatically in a day. An investor who rebalanced quickly reaped substantial benefits. In 1929 the stock market also fell dramatically, but an investor who rebalanced immediately (and thereafter) probably was soon eating at soup kitchens, as the market plummeted in subsequent years.”

[25] Israelsen, Craig, “Rebalancing Acts,” Financial Planning (June, 2001) pp. 59-62; and “Rebalance of Power,” Financial Planning (April, 2002) pp. 102-106.

operate under conditions of cash outflows. This means that terminal portfolio wealth (as well as, perhaps, interim consumption financed from the portfolio) is path dependent. Given the fact that a precise repetition of the sequence of past returns is highly improbable, past behavior may not be a good indication of future results. Additionally, there is the problem of creating an equal playing field for risk. A portfolio that produces more return for less risk dominates a portfolio that produces less return at greater risk. However, how does the trustee compare two portfolios when one has both greater return and greater risk? Under certain conditions, the return-per-unit-of-risk for each portfolio[26] can be evaluated to determine which portfolio has the superior risk-adjusted return. But there is the expectation that a portfolio allocated between stocks and bonds under a drifting-mix passive management regime will eventually approach a portfolio that is 100% stocks given a sufficiently long planning horizon.[27] Not only is the risk of a drifting mix portfolio much greater, the expected risk increases over time as equities dominate the asset weightings. Reward to risk ratios may not be the best comparative measure under these circumstances. This problem is known as "equity drift" and it presents a difficult task to the trustee wishing to make an apples-to-apples comparison of a static portfolio and a dynamic portfolio over time. There are a variety of ways to achieve "risk-calibration." Common adjustment methodologies involve simulation of returns to determine the average degree of equity drift or examination of historical portfolio positions to determine the average proportion of equity allocation. The *starting*

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[26] When return is defined as [(the excess return over the risk-free rate) ÷ (standard deviation of return)], the return-per-unit-of-risk is the well-known Sharpe Ratio. Sharpe ratios are comparable only when the investment periods are the same. That is to say, it is not possible to compare, on the same scale, variance over unequal holding periods.

[27] This is because the expected return from stocks is higher than the expected return from bonds or cash. However, the fact that the stock investment is riskier (has a greater dispersion in its returns) pushes the planning horizon out to amazingly long periods. Mark Rubinstein has a most interesting proof of this fact. He asks: "how long must an investor be prepared to wait before the probability becomes high that an all-stock portfolio will outperform an all-bond portfolio?" Rubinstein develops the following theorem: Assume that all available assets collectively follow a stationary random walk in continuous time (with finite variance). Let X and Y be the values after elapsed time t > 0 from following two strategies (with equal initial total investment), each being the result of continuously rebalancing a portfolio to maintain constant proportions in the available assets. Then:

$$\text{Probability } (X > Y) = N \left\{ \frac{(\mu_X - \mu_Y)\sqrt{t}}{[\sigma_X^2 - 2\rho\sigma_X\sigma_Y + \sigma_Y^2]^{1/2}} \right\}$$

where N is a joint standard lognormal probability distribution, μ_{Xt} is the expected value of log (X), $\mu_Y t$ is the expected value of log (Y), $\sigma_X \sqrt{t}$ is the standard deviation of log (x), $\sigma_Y \sqrt{t}$ is the standard deviation of log (Y), and ρ is the correlation between log (X) and log (Y).

Assuming, based on a reasonable sample of historical data, that stocks offer a 2.5% return premium over bonds, with the standard deviation of stocks equal to 18% and the standard deviation of bonds equal to 10% with a correlation of 0.4, in order to be 95% confident that an all stock portfolio will outperform an all bond portfolio requires a planning horizon of 123 years. Rubinstein, Mark "Continuously Rebalanced Investment Strategies," Journal of Portfolio Management (Fall, 1991). Trustees should keep Rubinstein's demonstration in mind when they are told to load up on stocks because "stocks have outperformed bonds over every 20 year period." History does not embody mathematical necessity; and, the non-satiation principle (more money is better than less) is a poor defense against allegations of breach of fiduciary duty to invest prudently.

equity weighting of the fixed-mix portfolio is adjusted to the *average* equity weighting so that risk exposures, "on average," are comparable[28]

Rebalancing Strategies

In terms of setting and preserving a trust's investment policy, commentators generally agree that rebalancing is an indispensable tool.[29] Why would a trustee be unwilling to rebalance a portfolio? There are several arguments against an active portfolio management process utilizing rebalancing transactions:

- Rebalancing costs money: trading is not free and, in some instances, trading costs can be formidable (trade commissions, market impact, bid/ask spreads, surrender charges, redemption fees, unamortized front end loads, etc.);[30]
- Rebalancing may trigger tax liabilities for taxable portfolios: this is an especially acute problem for poorly diversified portfolios invested in only a few assets with a low tax basis;[31] and,
- Rebalancing can truncate returns as money is taken away from winners and put into losers.[32] This problem may be especially acute in trending markets.

Rebalancing is, therefore, a process that balances transaction and tax costs against potential risk and return enhancements. A major argument for rebalancing is, in a nutshell, that it preserves the integrity of trust investment policy by adherence to the asset allocation guidelines. A major argument against rebalancing is that it costs money.[33]

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[28] An approach involving simulation of returns to determine the average degree of equity exposure from drifting portfolios is found in: Buetow, Gerald W., Sellers, Ronald, Trotter, Donald, Hunt, Elaine & Whipple, W.A., "The Benefits of Rebalancing," The Journal of Portfolio Management (Winter, 2002), p.26.

[29] See, for example, Goodsall, Bill & Plaxco, Lisa, "Tactical Rebalancing," Investment Management Reflections (First Quadrant, L.P., 1996): "A strategy that does not manage ongoing asset allocation…does not safeguard the integrity of the fund's investment policy" p. 3. See also, O'Brien, John, "Rebalancing: A Tool for Managing Portfolio Risk," Journal of Financial Service Professionals (May, 2006), p. 62: "A prudent investor's investment policy will outline both an asset allocation strategy and guidelines for rebalancing the portfolios when changes in market conditions occur." A general discussion of rebalancing policy and prudent investing is found in Knox, William T., "Portfolio Rebalancing," New Jersey Law Journal (September 11, 2006).

[30] Uniform Trust Code §805 Costs of Administration. "In administering a trust, the trustee may incur only costs that are reasonable in relation to the trust property, the purposes of the trust and the skills of the trustee."

[31] Restatement Third §227 Comment 'e' General requirement of caution: "This is consistent with the trustee's ongoing duty to monitor investments and to make portfolio adjustments if and as appropriate, with attention to all relevant considerations, including tax consequences and other costs associated with such transactions."

[32] An investor may have neither a well-defined financial goal (a generation-skipping trust's goal may simply be expressed in terms of growing wealth for a future generation), nor a well-defined planning horizon (e.g. a "dynasty" trust), nor a benchmark allocation calibrated to specific economic objectives and risk tolerances. Drifting away from the benchmark is, therefore, not a particular concern. Such an investor is said to be indifferent with respect to risk and may not value portfolio rebalancing actions. An additional utility-based argument against rebalancing is discussed later in the paper.

[33] Uniform Prudent Investor Act §2 Comment: "Under the present recognition rules of the federal income tax, taxable investors, including trust beneficiaries, are in general best served by an investment strategy that

Several academic studies focus on the topic of rebalancing rules. In general, rebalancing rules prescribe both formulae under which the investor initiates rebalancing, and the extent of the rebalancing actions. A review of the literature indicates that, in general, the formulae initiating rebalancing actions fall into one of five categories:

1. Calendar based rules: rebalance daily, monthly, quarterly, yearly, etc.[34];
2. Drift from initial asset allocation proportions (i.e., if a 10% proportionate drift triggers a rebalance action, for a two asset class portfolio with a 60% stock allocation, the investor would initiate rebalance transactions if the stock grew to 66% of the portfolio's value or diminished to 54% (±10% of 60%);[35]
3. Fixed percentage drift (i.e., if an asset class drifts more than ±10% of total portfolio value, the investor takes rebalancing actions. For example, given a 60% allocation to stock, if stock value increases to more than 70% weighting or decreases to less than 50% portfolio weighting, the investor would take rebalance action. In this category, the investor can set differing upper and lower bounds for each asset class);[36]
4. Standard Deviation criteria: rebalancing occurs when an asset's risk premium (return in excess of the risk-free rate) shows a marked increase or decrease over its historical mean;[37]
5. Other criteria: including continuous rebalancing using a derivatives portfolio overlay (futures contracts), probability-based rebalancing based on creating of value-added confidence intervals, and so forth.[38]

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minimizes the taxation incident to portfolio turnover." See also, Uniform Prudent Investor Act §7 Comment: "Wasting beneficiaries' money is imprudent. In devising and implementing strategies for the investment and management of trust assets, trustees are obliged to minimize costs."

[34] Arnott, Robert D., & Lovell, Robert M., "Rebalancing: Why? When? How Often?" Journal of Investing (Spring, 1993), p. 7, suggest that monthly rebalancing "…might add 41 basis points to the annual return of a 50/50 portfolio (9.16% versus 8.75%)." The gain is equal to increasing the allocation to equity by 11 percent. Yearly rebalancing added value that was almost equivalent to monthly rebalancing (9.02% versus 9.16%). Dennis, Patrick, Perfect, Steven B. & Snow, Karl N., "The Effects of Rebalancing on Size and Book-to-Market Ratio Portfolio Returns," Financial Analysts Journal (May-June 1995), pp. 47-57, conclude that the optimal rebalancing interval for a long-only portfolio is two years.

[35] Stine, Bert & Lewis, John, "Guidelines for Rebalancing Passive-Investment Portfolios," Journal of Financial Planning (April, 1992), pp. 80-86. Arnott, Robert D, Burns, Terence E., Plaxco, Lisa & Moore, Philip, "Monitoring and Rebalancing," Managing Investment Portfolios: A Dynamic Process 3rd edition, Eds. John L. Maginn , Donald L. Tuttle, Jerald E. Pinto & Dennis W. McLeavey (Wiley, 2007), pp. 701-716 discusses rebalance strategies with particular attention to factors determining the width of corridor limits for percentage of portfolio rebalancing strategies.

[36] Evensky, Harold, "Rebalancing Act: A formula for rebalancing to avoid style drift," Financial Planning (June 1996), pp. 170-171, 193).

[37] See, for example, Arnott, Robert D., & Lovell, Robert M., "Monitoring and Rebalancing the Portfolio," Managing Investment Portfolios: A Dynamic Process edited by John L. Maginn & Donald L. Tuttle (Warren Gorham & Lamont, 3rd edition), Chapter 13. Standard Deviation approaches can also be based on asset volatility or heteroskedasticity which triggers rebalance actions based on extreme movements in asset values.

[38] For a discussion of a derivatives overlay portfolio see: Buetow, et al., Supra. Probability-based rebalancing is the subject of a paper by Michaud, Robert O., and Michaud, Richard O., "Resampled

Recommendations regarding the extent of rebalancing (as opposed to the frequency dictated by the above-listed formulae), generally follow one of three recommendations:

1. Rebalance to the normal policy targets (i.e., for the fixed-mix portfolio rebalance to the asset allocation targets);
2. Rebalance to the boundary or threshold limit at which the transaction is triggered; or,
3. Rebalance to a point between the asset allocation target and the boundary.

Mathematical Studies: Rebalancing and the Algebra of Diversification

Mathematical treatments of portfolio rebalancing generally fall into two categories:

1. Examination of the "algebra of diversification;" and,
2. Examination of the cost/utility benefit tradeoffs of rebalance elections.

Often, when studying portfolio investment returns over time, it is important to distinguish between the return from the risk premium[39] of investment positions, and the return generated by rebalance strategies. Although somewhat counterintuitive, it is well known that portfolios for which the average investment has a risk premium close to zero, can generate substantial compound returns, over time, provided that the portfolio's investments exhibit certain statistical characteristics. These include high standard deviations for the individual investment positions as well as low correlations between the positions. For example, a recent study of commodity futures concludes that incremental portfolio returns are primarily generated by rebalancing strategies as opposed to investment risk premiums.[40] Mathematically, the authors define a "diversification return" as the difference between a portfolio's geometric (compounded over time) return and the weighted sum of the geometric return of each investment position held within the portfolio. This difference, in turn is decomposed into two parts: (1) a variance reduction benefit; and, (2) a covariance drag. Algebraically:

Diversification Benefit = (Variance Reduction Benefit - Covariance Drag).

The above formula is important with respect to rebalance strategy because the covariance drag term equals the economic impact of not rebalancing (i.e., following a drifting mix asset management approach). We consider each term in sequence.

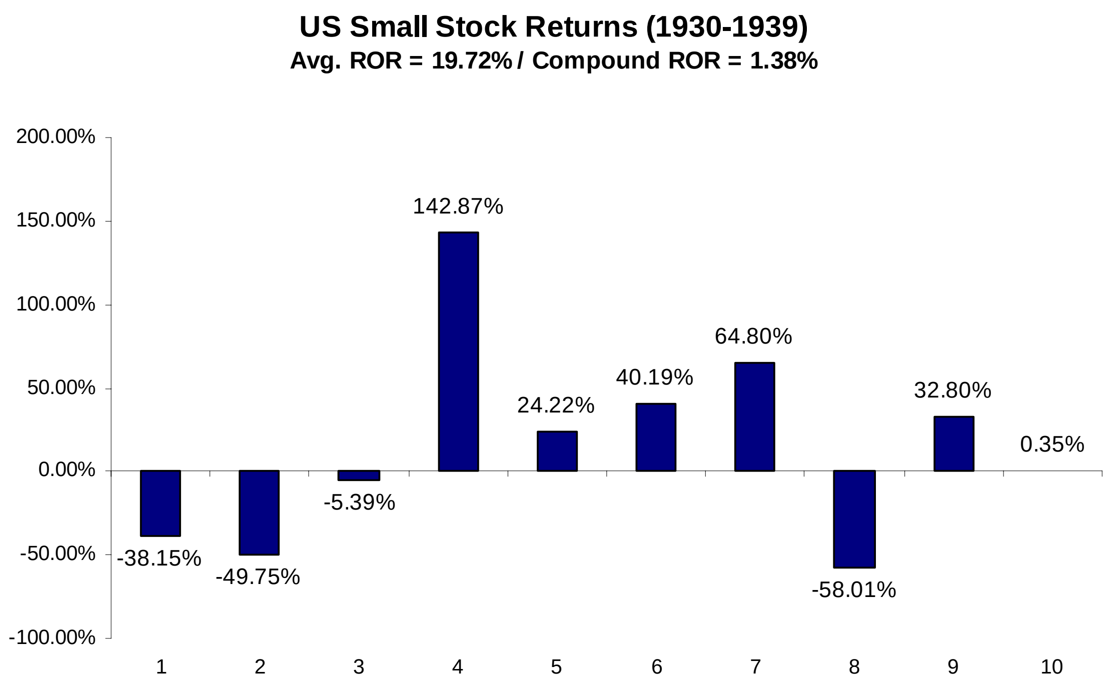
The variance reduction benefit is one of the most important reasons why trustees elect to diversify portfolios especially when they are asked to provide ongoing cash distributions to beneficiaries. Consider the following example: if a trustee invests $10,000 for thirty years at a compound return of 13%, the ending wealth amounts to $494,024. At an 11%

Portfolio Rebalancing and Monitoring," available at www.newfrontieradvisors.com. This paper is an extension of research originally published in Michaud, Richard O., Efficient Asset Management (Oxford University Press) 1998. Rebalancing to a "peer group" consensus benchmark is discussed in Lowe, Stephen, "Rebalancing the Portfolio," Asset Allocation in a Changing World (AIMR 1998), pp. 117-125.

[39] Where risk premium is defined as the risky asset's return in excess of the risk-free rate.

[40] Erb, Claude B. & Harvey, Campbell R., "The Tactical and Strategic Value of Commodity Futures," Working Paper Duke University (January 6, 2006).

return, ending wealth is $271,126. A 2% difference in compound return translates into a 45% difference in ending wealth. In this example, terminal dollar wealth tracks investment return (i.e. the higher the return the more dollars produced at the end). Return and terminal wealth line up because the 13% and 11% returns are constant (i.e. they show no variance). Constant returns are found in bank CDs, zero-coupon treasuries held to maturity, and similar financial instruments. The certainty in return, however, means that it is difficult to find high rates for such investments. Most investments offering higher returns, especially stocks, do not produce a constant return. For example, the following graph depicts a ten-year period during which returns from US small company stocks generated a 19.72% average yearly return:



Although the average annual return is almost 20%, the return series produced a compound rate of wealth accumulation of only 1.38% despite the fact that there was one year during which the investment earned a 142.87% return.

The difference between average return and compound return is the subject of a research paper that poses the question "Is your investment return leaking down the variance drain?" [41] The author points out that investors spend *dollars* not *rates of return*. Variance in investment returns subtracts from ending dollar wealth according to the following mathematical approximation:

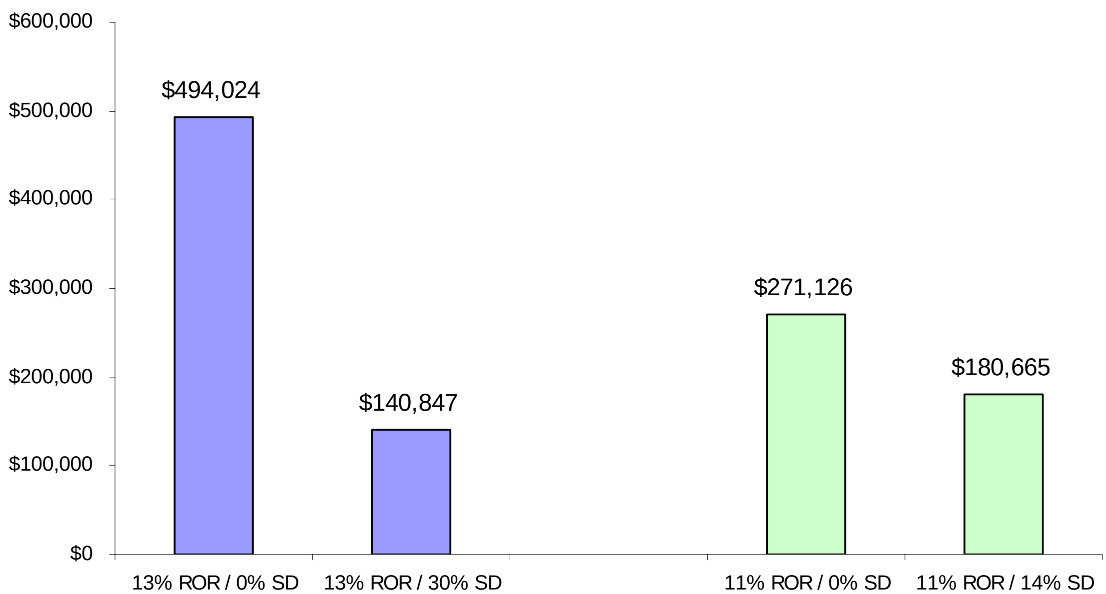
Ending Compound Wealth = Initial Wealth x [Average Return - ½ Variance].

Variance, in turn, is the square of standard deviation. Standard deviation measures how far actual period-to-period investment returns differ from the average return during the entire period. For example, since 1970, the average US stock has earned approximately

[41]Messmore, Tom, "Variance Drain," The Journal of Portfolio Management (Summer, 1995).

13% per year with a standard deviation of 30%. This can be contrasted with a portfolio (combinations of individual stocks) that earns 11% with a standard deviation of 14%. As the following graph indicates, when the variance drain factor is included in calculations of ending wealth, the investor achieves better results under the 11% earnings regime. Initial value equals $10,000 invested over a 30 year planning horizon:

Effect of Variance Drain on Wealth Accumulation



If the variability (uncertainty) of return is too great, even spectacular percentage rates of returns will, in the end, produce only modest wealth. Variance drain is easy to demonstrate mathematically. If an investor puts $1 into each of two investments with equal average return but different volatility, the end results will not match. If for, example, investment A earns –10% year one and +10% year two, its average return is 0% and its ending wealth is $1(.9)(1.1) = $0.99. If investment B earns –20% year one and +20% year two, its average return is also 0%. Its ending wealth, however, is $1(.8)(1.2) = $0.96. Absent cash flows, the order in which the returns are achieved is irrelevant. The important point is that the greater the variability in returns the lower the investor's spendable wealth. However, trustees can distribute only dollar wealth; therefore the mathematics of variance drain has critical significance for trust beneficiaries.

When faced with the obligation of making periodic income distributions to the current beneficiaries, the order in which returns occur matters very much. When a portfolio makes periodic distributions, such distributions act as a multiplier of downside results[42]

[42] A condition sometimes termed 'feeding the bear.' Weiss, Gerald R., "Dynamic Rebalancing," Journal of Financial Planning (February, 2001). Weiss recommends a rebalance strategy that, during periods in which equity performs poorly, portfolio withdrawals are financed primarily from fixed income portfolio investments (no equity is sold to fund distributions); otherwise, withdrawals and portfolio rebalancing objectives are achieved by annually rebalancing to a target allocation. This dynamic strategy is compared

and as a cap on upside results. Following a period of negative return, the act of removing dollars from the portfolio means that fewer dollars remain to assist in the recovery of wealth; following a period of positive return, the act of removing dollars from the portfolio means that fewer dollars remain available to continue compound growth. Variance drain can erode portfolio wealth despite the fact that all investment risk/return parameters operate according to the money manager's forecasts.

Here is a highly stylized example: we know (because multiplication is commutative) that 3*2*1 = 6; and that 1*2*3 = 6. The order of the 'returns' does not matter. This principle holds for any compound return series in which there are no cash flows. Consider, however, what happens when we add cash-flow requirements to the series:

- Period One: 1*3 = 3 - ½ = 2½ *2 = 5
- Period Two: 5 - ½ = 4½ * 1 = 4½
- Period Three: 4½ - ½ = **4 units of ending wealth.**

However,

- Period One: 1*1 = 1 - ½ = ½ * 2 = 1
- Period Two: 1 - ½ = ½ * 3 = 1½
- Period Three: 1 ½ - ½ = **1 unit of ending wealth**.

The order in which returns are earned matters when there are cash flows. An average return "target" is no longer sufficient in the presence of portfolio distribution requirements and return variance. Cash flows change the definition of 'required return,' which, in turn, may change both the trustee's asset allocation preferences and rebalance strategy. A key element in skilled asset management is the trustee's ability to coordinate the portfolio's asset allocation policies, rebalance strategy and distribution requirements. This suggests that trust investment policy should be 'n'-dimensional rather than a one-dimensional hunt for attractive rates of investment returns.

If there is a benefit to variance reduction, so also there is a cost for forsaking the opportunity to rebalance. Erb and Harvey term the cost of not rebalancing 'covariance drag:' "…on average the portfolio weights of individual assets in an unrebalanced portfolio covary negatively with the returns of the individual assets. This results in a negative impact of not rebalancing."[43] The authors quantify the cost of not rebalancing for a hypothetical equally-weighted portfolio of 40 uncorrelated securities each of which have a zero expected risk premium and a standard deviation of thirty percent. They simulate 10,000 45-year return histories for each of the 40 securities, for a portfolio that is never rebalanced, and for an equally weighted portfolio that is rebalanced annually. On average, the individual securities generated an average annual return in excess of the risk free rate close to zero. The compound return of the rebalanced portfolio (4.3%) exceeded the compound return of the drifting portfolio (3.8%) by 50 basis points per

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to both a straightforward inflation-adjusted distribution policy and a "keep an emergency reserve" to buffer market volatility policy; and is found to be the superior alternative.

[43] Erb and Harvey, Supra, p. 60.

year. The rebalanced portfolio generated more ending wealth in 71% of the simulations and had a better reward-to-risk tradeoff (Sharpe Ratio) in all 10,000 trials. In this simulation, the cost of not rebalancing (i.e., the covariance drag) equals 50 basis points per year.

The authors derive the following equation to estimate the diversification return of an equally weighted rebalanced portfolio of uncorrelated assets where each asset's returns are not serially correlated:[44]

$$\text{Diversification Return} = \frac{1}{2}\left[1-\frac{1}{K}\right]\overline{\vartheta}\left[1-\overline{\rho}\right]$$

Where compound return increases with the number of securities in the portfolio (K); increases with the average standard deviation of the securities (σ); and, increases as the average correlation of securities diminishes (ρ). They indicate, "when asset variances are high and correlations are low, the diversification return can be very high." However, "…positive autocorrelation of returns might lower… diversification return."[45]

Although rebalancing strategies are important tools to preserve the integrity of the trust's investment policy, the algebra of diversification provides additional justifications for a trustee to consider the design and implementation of portfolio rebalance policy:

1. Estimation of future returns from a rebalanced portfolio is relatively straightforward in that the calculations require only a forecast of expected return, variance and correlation coefficients. Estimation of future returns from unrebalanced portfolios, by contrast, is difficult. The impact of not rebalancing must be included in the estimation; but this impact is path dependent.
2. In some cases, given the statistical characteristics of the investments within the trust portfolio, unrebalanced portfolios may outperform portfolios that are periodically rebalanced. In such cases, however, the outperformance may not be due to greater investment efficiency; but merely to the increased risk associated with equity drift. Voluntarily assuming risk that is unnecessary in terms of the trust's terms, purposes, distribution objectives and other circumstances may be imprudent.

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[44] Serial correlation is present when an asset's returns are correlated with each other over successive time intervals. Serial correlation is also termed autocorrelation—the correlation of a security with itself.

[45] In a separate paper, Bernstein, William J., "The Rebalancing Bonus: Theory and Practice," Efficient Frontier (1996) http://www.efficientfrontier.com argues: "only when long term return differences among assets exceed 5 percent do nonrebalanced portfolios provide superior returns, and then only at the cost of increased risks." Bernstein derives the following formula for estimating the rebalancing benefit for a two asset portfolio:

$$X_1X_2(Var_1/2 + Var_2/2 - Covar_{1,2})$$

Where 'X' is the weight of the asset within the portfolio, 'Var' is the variance of returns; and 'Covar' is the covariance of returns. The Bernstein formula takes advantage of the well-known statistical identity: $Covariance_{1,2}$ = (Standard $Deviation_1$)(Standard $Deviation_2$)($Correlation_{1,2}$). Thus, the Bernstein formula has a term for variance, a term for correlation and a term for covariance drag.

Mathematical Studies: Maximizing Expected Utility

Hayne Leland at the University of California, Berkeley, authored a seminal mathematical study of rebalance tradeoffs.[46] Drawing upon earlier studies by George Constantindes, and B. Dumas & E. Luciano,[47] Leland's study argues that when rebalancing costs are proportional to the amount rebalanced,[48] the optimal strategy involves:

1. Placing a "no-trade" zone around the asset; and,
2. If the proportional weighting of the asset within the portfolio penetrates the no-trade barrier, rebalancing should be undertaken to restore the asset weighting to the nearest edge of the barrier rather than to the asset allocation target.

Leland does not specify a particular utility function for the investor, but develops a cost function for deviations from the targeted asset allocation. If the target allocation is well synchronized to the investor's wealth accumulation and consumption objectives, deviation from the target will result in a measure of "disutility" to the extent that it makes the attainment of future economic goals more uncertain.[49]

Leland's model uses two asset classes (stocks = S and bonds = B) that follow log random walks (Brownian Motion). Therefore, according to the fundamental law of the evolution of wealth, the instantaneous rate of change for the stock position equals

$$\frac{dS(t)}{S} = \mu_S dt + \sigma_S dZ_S(t)$$

and, for the bond position, equals

$$\frac{dB(t)}{B} = \mu_B dt + \sigma_B dZ_B(t)$$

These equations state that the change in wealth follows a process dependent on three factors: the expected return (μ = mean or average return); the expected standard deviation of return (σ = standard deviation) and an adjustment to the expected standard deviation (Z = a random process) that is characterized by a zero mean and unit variance (i.e., a

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[46] Leland, Hayne E., "Optimal Asset Rebalancing In the Presence of Transaction Costs," Working Paper No. RPF-261 Walter A. Haas School of Business Research Program in Finance Working Paper Series (August, 1996).

[47] Constantinides, George, "Capital Market Equilibrium with Transactions Costs," Journal of Political Economy (1986), and Dumas, B., & Luciano, E., "An Exact Solution to a Dynamic Portfolio Choice Problem Under Transactions Costs," Journal of Finance (1991), pp. 577-596.

[48] Transactions costs can be fixed (i.e. irrespective of the amount bought and sold the investor has only a fixed charge) or proportional. If proportional costs are, for example, one percent, a trade involving $1,000 of assets will cost $10 while a trade involving $1,000,000 of assets will cost $10,000.

[49] It is important to note that "target" in this context can have several meanings. In terms of a fixed mix approach to asset management, target means the asset allocation established in the trust portfolio's investment policy; in terms of an insured portfolio approach, target means the application of the multiplier to the portfolio's equity surplus. In many studies, the term "rebalancing" refers to all portfolio changes required to reoptimize the portfolio to reflect changing trust objectives, economic circumstances, and investment forecasts. Rebalancing, according to this use of the term, is akin to 'recalibrating' the portfolio's design to maximize utility over the forthcoming period. Deviations from the optimal portfolio (i.e., tracking error) create disutility.

random draw from a standard log-normal distribution). The mean sets the general direction for the evolution of the vector of wealth over time, but the evolution of wealth is uncertain because the portfolio's volatility generates a range of possibly negative and positive values for each vector component (period return).

The ratio of stocks to bonds will determine the future wealth of the investor. Thus, if we define wealth by the letter 'w,' the instantaneous rate of change in the investor's wealth equals:

$$\frac{dw(t)}{w} = (\mu_S - \mu_B + \sigma_B^2 - \rho\sigma_B\sigma_S)dt + \sigma_S dZ_S(t) - \sigma_B dZ_B(t)$$

where
ρ = the correlation between stocks and bonds, and,
t = the applicable interval of time.

As stated, any difference between the target portfolio asset allocation (presumed to be optimal for the needs, goals and circumstances of the trust) and the actual portfolio will generate a loss of utility. Utility loss (L) is measured by the degree of tracking error [the squared deviations of the actual portfolio (w(t)) from the optimal portfolio (w*)], with the utility cost-per-unit-of-tracking-error measured by the parameter lamda (λ):

$$\text{Utility Loss} = L = \lambda(w(t)-w^*)^2 dt.$$

Over the applicable planning horizon, the investor will want to minimize the discounted (present valued) integral (sum) of the cost of divergence (λ) plus any trading costs associated with the rebalancing function. Leland's model is "self-financing" in that any costs associated with rebalance trades are paid from without rather than from within the portfolio. This makes the mathematics more tractable because any change in the value of the stock allocation (δS) is offset exactly by a change in the value of the bond position with the change having the opposite sign (δS = -δB).

A brief review may be helpful. Leland posits a "no trade" region around the stock and bond positions because the costs of rebalancing small variations from the optimal portfolio may be greater than the cost in "disutility" by not maintaining the strict asset allocation targets of the optimal portfolio. Costs, therefore, involve two terms: a monetary term (κ) measured by trading costs and a utility term (λ) measured by the increased uncertainties of not adhering to the asset allocation target. The total cost function, therefore, includes κ+λ with κ equal to $0.00 whenever the asset drift remains within the bounds of the no trade region (w_{max}, w_{min}). Putting it all together, the investor wishes to achieve the most favorable total cost function (V) over the integral of all periods within the applicable planning horizon:

$$V(w(t); w_{max}, w_{min}) = E\{[\int_{\tau=t}^{\infty} e^{-r(\tau-t)} \lambda(w(\tau) - w^*)^2 d\tau + PV\{transactions\ costs\}]\ given\ w(t)\}$$

This equation says that investor utility is maximized when the present value costs of tracking error plus the present value costs of rebalance transactions are minimized for any given level of wealth.[50]

After solving a complex differential equation,[51] Leland's monograph continues by calculating the optimal rebalance strategy. According to the model, it is optimal to return the portfolio's stock weighting to the edge of the no trade boundary rather than to the 60% allocation target; and Leland compares the costs and estimated turnover of this strategy with a quarterly calendar-based rebalance formula that maintains the exact allocation target. He estimates a cost savings of approximately 50% when the investor uses the optimal strategy.[52] Needless to say, if there are other costs (i.e., taxes), the width of the no-trade region expands: "In many cases, the no-trade interval changes with the cube root of the parametric changes." [53]

Tactical Rebalance Strategies

Many recent mathematically-oriented studies extend model building to encompass a greater range of complex issues—tax costs, a wider variety of risk aversion/utility of wealth functions, the introduction of non-liquid assets into the portfolio (e.g., hedge funds, illiquid annuities, and so forth), and non-Gaussian return distributions [54] A study published by Lynch and Balduzzi[55] finds that realistic transaction costs:

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[50] Leland's argument also assumes that a passive portfolio management strategy (i.e. drifting mix) has a quantifiable cost that can be measured relative to the ideal asset allocation. Thus, rebalancing mitigates "tracking error." This has proved to be a fruitful concept in much recent research. For example, one study notes: "What drives the benefit of rebalancing is reducing the tracking error from getting far off-target. As it happens, tracking error is quadratic. It's proportional to the square of the deviation from the target allocation. For example, when a portfolio with a 30% target for U.S. bonds gets to 32% bonds (2% over target) the tracking error is four times as high as being 1% over target. And if bonds climb to 33% of the portfolio, the tracking error risk is nine times the risk associated with a 1% deviation." Masters, Seth J., "Rules for Rebalancing," Financial Planning (December, 2002), pp. 89-93.

[51] Maximizing the first two derivatives of the total value function.

[52] Although trading frequency may increase, Leland's model assumes that trading costs are strictly proportional and not fixed. Additionally, the amounts traded will be very small. Small trades generate only small costs (unlike the fixed or step-rate trade commission schedule found at most brokerages) and all "costly trades" within the no-trade zone (i.e., trades that produce more trading costs than investor utility increases) are eliminated. The model's predicted cost savings may not be attainable under most wealth administration platforms in today's marketplace.

[53] Other studies indicate that the nature of the costs (i.e., fixed, proportional, or a combination thereof), determine whether the rebalance action should bring the asset weighting to the closest boundary of the no-trade region or to a point inside the region. Yip, Kenneth & Donohue, Christopher, "Optimal Portfolio Rebalancing with Transaction Costs," The Journal of Portfolio Management (Summer, 2003), pp. 49-63 provides an insightful extension of the Leland two-asset class model to multi asset class portfolios. In addition to examining the nature of the no-trade-region in the face of changing transaction costs, risk aversion, and correlations, they simulate various rebalance strategies. They recommend using a no-trade-region approach to rebalancing; but conclude "…the complexity of optimal rebalancing quickly makes it intractable, thus limiting its application to simple portfolios…."

[54] The nature of the return distribution is an important determinate of the suitability of a portfolio management strategy. Normal distributions or random walks are good candidates for rebalancing because the rebalance actions do not affect the expected future portfolio returns. As noted, rebalancing actions

- Cause the rebalancing frequency to decline considerably;
- Do not materially influence the amounts allocated to risky assets (which, presumably, would trigger more frequent rebalancing because of their relatively high variance); but,
- Lowers the amount allocated to risky assets if, in addition to rebalance costs, the risky assets have low liquidity.[56]

Additionally, the Lynch/Balduzzi study has important implications for rebalancing under tactical asset allocation regimes. Specifically, when returns are predictable, the no-trade interval widens considerably and the investor is more willing to incur transaction costs when the boundaries are penetrated. Holding unconditional return parameters (expected return, variance and higher moments of the distribution) constant, the Lynch/Balduzzi model explores an investor's rebalancing behavior given both the magnitude of single-period predictability and the persistence of predictability over several time periods. Lack of persistence, the authors find, causes the no-trade region to change from a band or straight-line boundary to a U shaped region. Furthermore, investors are more comfortable holding risky assets at wider trade ranges when faced with conditional distributions.

Tactical asset rebalance approaches remain controversial. A tactical approach takes the opportunity to blend periodic or threshold rebalancing elections with the manager's 'market timing' views concerning relative capital market valuations. Jensen and Mercer advocate rebalancing the portfolio to reflect turning points in the business cycle where such points are proxied by changes in the discount rate set by the Federal Reserve Board's Open Market Committee.[57] Goodsall and Plaxco advocate adjusting asset allocations tactically on either side of the strategic benchmark allocation.[58] Such a strategy constrains tacking error while providing the opportunity to add alpha by refraining from rebalancing when forecasting models indicate trending market conditions; or increasing rebalance frequency if models indicate the likelihood of volatile markets.[59] Smith and Desormeau provide a comprehensive examination of various rebalance formulae operating over a wide range of asset allocations during the period 1926 through 2003.[60] They conclude that, over most allocations, a patient approach to

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within mean-reverting distributions should add substantial value to the portfolio as the investor sells high and buys low. Rebalancing into trending markets, however, can drain money from the portfolio.

[55] Lynch, Anthony W., & Balduzzi Pierluigi, "Predictability and Transaction Costs: The Impact on Rebalancing Rules and Behavior," The Journal of Finance (October, 2000), pp. 2285-2309.

[56] Inclusion of non-liquid assets like hedge funds into an investment portfolio cause material difficulties for many rebalance approaches.

[57] Jensen, Gerald, R. & Mercer, Jeffrey M., "New Evidence on Optimal Asset Allocation," Financial Review (August, 2003), pp. 435-454.

[58] Goodsall & Plaxco, Supra.

[59] There is an extensive literature on the subject of volatility estimation. A seminal paper on this topic attempts to model volatility by taking advantage of its tendency to exhibit serial correlation. Bollerslev, Tim, "Generalized Autoregressive Conditional Heteroskedasticity," Journal of Econometrics (1986), pp. 307-327. Technical demands for this type of forecasting are, in general, beyond the skill set of most amateur trustees and of many professional trustees.

[60] Smith, David M. & Desormeau, William H., "Optimal Rebalancing Frequency for Bond/Stock Portfolios," Journal of Financial Planning (November, 2006).

rebalancing, which takes its cues from changes in Federal Reserve monetary policy, is optimal.

A key risk of tactical asset rebalance strategies is 'model risk'—that is the degree to which even a well-specified market-forecasting model may yield spurious results. With regard to model inputs, much, although not all, academic evidence suggests that it is both difficult to find predictor variables; and, when found, such variables may cease to exhibit predictive value in future economies. With respect to the model itself, although many types of models are compatible with financial data, their forecasts may differ significantly. In a nutshell, even assuming the existence of valid forecasting model inputs, it remains difficult to form profitable money management strategies at a high level of confidence.[61] For example, Handa and Tiware co-authored a study examining the economic advantages realized by an informed investor relative to a naïve investor.[62] Investor one is "uninformed" in the sense that he bases decisions on the unadjusted historical distribution of market risk and return. They term this investor the "i.i.d." investor, which is shorthand for the statistical phrase "independent and identically distributed." Such a distribution is characteristic of a normal (bell curve) distribution of a random variable such as the sum of coin flip results. Assuming a fair coin, each result is independent of all previous results, and there is no period in which it is more likely to flip either heads or tails. Investor two, by contrast, is a professional, "informed" investor that uses predictive (conditioning) variables to formulate beliefs regarding the future evolution of stock prices. They term this investor the "mutual fund" investor. The mutual fund investor uses combinations of three conditioning variables: (1) lagged stock returns to exploit information regarding serial correlation of time series; (2) lagged dividend yields to take advantage of earnings based information; and (3) book-to-market price ratios to capture information in relative valuations. The mutual fund investor continuously updates (on a quarterly basis) the parameters of predictive models so that he may capture the dynamics of the unfolding stock price process. However, the mutual fund investor is only permitted to use data that was actually available at the time in which an investment decision is made.[63]

The questions of primary interest to trustees contemplating a tactical rebalancing approach are (1) do the predictor variables allow the informed investor to generate *future* gains which are both statistically and economically significant when compared to those generated by the i.i.d. investor; (2) are the excess returns stable over time; and, (3) do the gains persist after expenses. Results are expressed as units of investor utility with each investor exhibiting quadratic utility at various risk aversion values. The i.i.d. investor

[61] Malkiel, Burton G., "Can Predictable Patters in Market Returns be Exploited Using Real Money?" The Journal of Portfolio Management (30th Anniversary Issue, 2004), pp. 131-141.
[62] Handa, Puneet & Tiware, Ashish, "Does Stock Return Predictability Imply Improved Asset Allocation and Performance?" Working Paper, University of Iowa (2000).
[63] The authors select the three conditioning variables based on a rich history of academic research into the predictive content of numerous accounting, fundamental, and macroeconomic variables. Although some trust companies and investment consulting firms produce performance reports with page after page of data charting trends in industrial production, inflation trends, growth in gross domestic product, energy prices, consumer confidence surveys, and so forth, the data's primary value is explanatory rather than predictive—a fact that is sometimes not made sufficiently clear to the report reader.

optimizes utility by developing a portfolio that uses only the sample mean return and the sample covariance matrix, which are both updated each quarter. The informed mutual fund investor optimizes utility by forming portfolios based on *predicted* returns by employing a more complicated set of factors and factor loadings.[64]

The paper presents a matrix of test results. Tests are based on portfolios formed from either a single risky asset or from multiple risky assets, on investors with various risk aversion parameters (low, moderate and high), on various combinations of the predictor variables, on various methods of adjusting for uncertainty in parameter values, and over various sub periods using market data from January 1954 through December 1998. In each test, the performance of the i.i.d. investor's portfolio is compared to that of the mutual fund investor's portfolio in terms of its Sharpe (Reward to Risk) Ratio and to the Certainty Equivalent measure (mean return - ½ variance x risk aversion coefficient). In order to mitigate the problem of data snooping and to determine if results are robust, tests are based on simulations of bootstrapped (resampled) data. The authors test on out-of-sample data by allowing each investor to use a five-year learning period prior to forming portfolios. Portfolios are then evaluated on a go forward basis.

The major findings are:

- Under most tests, the predictor variables are unstable over time and have only period specific value;
- Performance gains, if any, realized by the mutual fund investor are not robust and are unlikely to persist into future periods;
- Combinations of two or more predictor variables do not improve the mutual fund investor's performance;
- Utilizing predictor variables does not result in an increased ability to time markets ("…the market timing ability of the mutual fund strategy is fairly dismal.").

It is noteworthy that the performance of the mutual fund investor significantly lags that of the i.i.d. investor over the most recent sub period (1985 through 1998).

Historical Returns or Adjusted Returns?

George Santayana's familiar phrase "Those who cannot remember the past are condemned to repeat it" is often employed by armchair philosophers to draw parallels between current circumstances and past events. Unfortunately, in financial economics, the past may not be particularly relevant for forecasting the future. The empirical literature discussing the costs and advantages of rebalancing, usually assumes that the single path of realized historical returns is sufficient to derive trustworthy parameters regarding the distribution of future asset returns. Designing a portfolio rebalance strategy

[64] Each investor uses the updated but unadjusted sample covariance matrix. The mutual fund investor updates using a Bayesian approach where the predicted returns are conditional on the information set.

based on data mining of historical returns, however, may be a suboptimal approach. This is the case for at least two reasons:

1. Cash flows may create path dependencies. This means that the order in which returns are earned matters at least as much as the average and variance statistics that summarize the return distribution. Unless the sequence of past returns repeats exactly—a highly improbable prospect—the relevance of the single pattern of realized past returns is limited. Good decision making under such conditions is a function of analyzing many possible return paths over a large number of simulated future economies.

2. Historical returns manifest a degree of dominance by certain asset classes that would not be expected under conditions of equilibrium.[65] For example, the most well known equilibrium asset-pricing model—the Capital Asset Pricing Model [CAPM] assumes that all assets should have similar risk-adjusted return. The market offers little opportunity to generate net present value profits because assets with higher nominal returns must be discounted more heavily for their risk. This raises an issue concerning the degree of confidence that the investor is willing to put in the informational content of historical data.

The extent to which an investor is willing to trust historical returns will affect the of asset allocation decision as well as the choice of a suitable rebalance strategy. A dogmatic reliance on historical return estimates suggests that, in some cases, investors may wish to curtail or refrain completely from rebalancing out of certain historically dominant asset classes.[66] Indeed, at the limit, a strict historical point of view may suggest that it is prudent to place all portfolio wealth in the single dominant asset class.

Taxes

Tax issues further complicate the rebalance decision.[67] One area of complexity is the differential tax rates or differing tax treatment of ordinary income (dividends vs. interest), short-term gain or loss, and long-term gain or loss.[68] Rebalance actions that harvest

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[65] Equilibrium in a financial economics context assumes that investors maximize their unique utility functions in such a way that prices of financial assets equilibrate the supply of securities with the demand for securities.

[66] Discussion of uncertainty in parameter estimates derived from historical return series is well beyond the scope of this essay. However, it should be noted that, for many historical return series, calculation of mean (average) return is subject to a large standard error of estimate (a phenomenon known as 'mean blur') and requires very lengthy return histories

[67] The following discussion draws upon Horvitz, Supra, pp. 49-53. For a study of the interrelationships between asset allocation, consumption and rebalancing strategies see, Hughen, J. Christopher, Laatsch, Francis E., & Klein, Daniel P., "Withdrawal Patterns and Rebalancing Costs for Taxable Portfolios," Financial Services Review (Vol. 11, 2002), pp. 341-366. Weinstein, Steven B., Sin-Yi Tsai, Cindy & Laurie, Jason M., "The Importance of Portfolio Rebalancing in Volatile Markets," CCH Retirement Planning (July/August, 2003), pp. 37-42, incorporates tax assumptions in an empirical study of a 60 percent stock / 40 percent fixed income portfolio. The study uses an annual rebalancing schedule for returns during the 1980 through 2002 period. It employs actual federal marginal tax rates for both ordinary income and capital gains; and, it applies these rates to an assumed taxable turnover rate. The study found that annual rebalancing increased the portfolio's after-tax returns by 20 basis points per year; and reduced annual standard deviation from 11.6% (for the drifting portfolio) to 10.6% (for the rebalanced portfolio).

[68] Not to mention, the propensity of the U.S. Congress to change tax law with great frequency.

losses as opposed to those that recognize gains may be more readily taken.[69] Gain recognition destroys the time value of tax deferral which is a form of opportunity cost for the investor. Given today's low tax rates, however, the value of the tax deferral may not be great enough to justify portfolio concentration risk through retention of low-basis assets. One measure of the value of the tax deferral is the amount of tax-payment on the recognized gain discounted by the tax-free investment rate for the applicable planning horizon. For example, an asset sale that triggers a $1 million long-term gain generates (at a combined 20% state and federal cap gains rate) a tax liability of $200,000. But the gain is an embedded gain that must, absent a sale, be paid at some future date (other than a sale after the date of the owner's death). The value of the tax liability deferral for a five-year period is, therefore, the present value of a future payment of $200,000 discounted by the five-year muni-bond rate. Assuming a tax-free rate of 3%, the value of the deferred payments equals $172,518. Thus the tax cost of the transaction equals ($200,000 - $172,518), or $27,482. When a $27 thousand tax cost on sale of a $1 million position (assuming a zero basis) is weighed against the risk of maintaining an underdiversified or misallocated portfolio, the costs of not rebalancing may well outweigh the acceleration of the tax liability.[70]

Prudence, Utility Theory, and Portfolio Rebalancing

Under what circumstances might it be prudent to drift? One possible answer to this question is based in utility theory. Most investors prefer more wealth to less wealth, but have a decreasing marginal rate of satisfaction. Earning an additional dollar produces slightly less satisfaction than losing a dollar produces dissatisfaction. Most investors are sensitive to changes in their dollar wealth (they exhibit "risk aversion") such that they exhibit increasing utility of wealth curves with positive first derivatives and negative second derivatives (head upwards but at a constantly decreasing rate). There are a variety of curves that can fit into the generalized mathematical model known as von Neumann-Morgenstern utility functions.[71]

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[69] Berkin, Andrew L., & Ye J., "Tax Management, Loss Harvesting, and HIFO Accounting," Financial Analysts Journal (July/August, 2003), pp. 91-102.

[70] This simplified example ignores the uncertainty of future returns on the value of the underlying portfolio assuming no transaction. For a more complete discussion, see Stein, David M., "Diversification of Highly Concentrated Portfolios in the Presence of Taxes," Investment Counseling for Private Clients II (AIMR Conference Proceedings, 2000), pp. 18-25; and Stein, David M., & Narasimhan P., "Of Passive and Active Equity Portfolios in the Presence of Taxes," The Journal of Private Portfolio Management (Fall, 1999), pp. 55-63.

[71] Von Neuman and Morgenstern explored the mathematics of choice in uncertain situations, and developed a series of axioms that characterize rational choices. For example, if the utility of x is preferred to y, and the utility of y is preferred to z, then the utility of x must be preferred to z. In an investment framework, the expected utility of wealth E[U(W)] is denoted as follows:

$$E[U(W)] = \sum p_i U(w_i)$$

where 'p' represents probability of a specific economic regime 'i,' and 'w' represents the amount of wealth generated by the portfolio in state 'i.' The sum of the probabilities are equal to one, thus the probability-adjusted utility of wealth equals the utility value of wealth generated under a variety of economic conditions (a dollar in a recessionary economy may have a greater utility than a dollar in an prosperous economy) adjusted for the probability of the occurrence of a specific economic state (recession, stagflation, growth, deflation, etc.). Harry Markowitz draws upon Von Neuman and Morgenstern's work but assumes

Two curves that fit nicely into the von Neumann-Morgenstern family are a logarithmic curve (Utility = log of wealth) and a quadratic curve (Utility = square root of wealth). The investor with the logarithmic utility curve is said to be more sensitive to changes in wealth because the curvature produced by the function is greater. Chances are that the portfolio that will best satisfy the log of wealth investor will not be the portfolio that will best satisfy the quadratic utility investor. The reason for this preliminary conclusion lies in the fact that these investors have very different views about risk and reward. A few investors have utility curves that are straight line (linear with respect to wealth). These investors are 'risk-neutral.' Others exhibit "gamblers' curves.". For gamblers, the thrill of the wager often has greater utility than the level of expected wealth offered by the wager's payoff.

Quadratic utility assumes that the investor becomes more risk averse as wealth increases. This means that the investor exhibits Absolute Risk Aversion [ARA] because a 5% negative return causes a millionaire to lose more dollars than a 5% negative return causes a child with a $100 Christmas club account to lose. Although this type of utility curve may seem counterintuitive, nevertheless, it is characteristic of certain groups of investors.[72] Many investors have utility curves that exhibit decreasing absolute risk aversion and constant relative risk aversion [CRRA].[73] Finally, there are investors with "kinked" risk aversion curves. They may feel comfortable with investment risk above a specified portfolio value (an investment "surplus"), but may become highly risk averse below the threshold value.[74] The important point to note is that some investors are highly

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that investment distributions are normal distributions fully determined by their risk (standard deviation) and expected return. Investors are, therefore, assumed to have quadratic utility—that is to say, only two elements (mean and variance) influence their investment choices. Mean is a first order term and variance is a squared term; hence, utility (U) can be represented by a quadratic equation $U = a\mu + b\sigma^2$. Using the Von Neuman and Morgenstern axioms, Markowitz derives the following utility functions:

- If the utility of a guaranteed result is greater than the expected utility of wealth over a variety of possible but uncertain outcomes (the expected return of which equals the certain return) {U[E(W)] > E[U(W)]}, then the investor is risk averse;
- If the utility of expected wealth equals the expected utility of wealth {U[E(W) = E[U(W)]}, then the investor is risk neutral; and,
- If the utility of expected wealth is less than the expected utility of wealth {U[E(W) < E[U(W)]}, then the investor is risk seeking.

Markowitz assumes that rational investors are risk averse. In the Markowitz world, the investor with quadratic utility prefers a certain $400 (utility = square root of 400 = 20) to a risky proposition offering, at 50/50 odds, either $100 (utility = square root of 100 = 10÷2 = 5) or $700 (utility = square root of 700 = 26.46÷2 = 13.23). Despite the fact that the uncertain proposition has the same expected value as the sure thing, the certain result is preferred because of its higher utility: 20 is greater than 5 + 13.23, or 18.23.

[72] One thinks of a business owner who takes much risk (puts all his eggs in one basket) building the *commercial* enterprise. At time of sale, however, the entrepreneur may exhibit great aversion to taking *investment* risk. Investors (and trustees) sometimes confuse requirements for commercial success (asset concentration) with requirements for investment success (asset diversification).

[73] Such curves are consistent with investors that exhibit log utility of wealth. Constant risk aversion means that both the millionaire and the child with the Christmas club account would be willing to risk 5% of their wealth given a reasonable expectation of investment gain. Most mathematical models of portfolio choice assume that investors have CRRA curves.

[74] This behavior is consistent with defined benefit pension plans or certain endowments and foundations that seek to maintain a plan surplus. A comparable set of behavior may be exhibited by an individual who

affected by shifts in wealth while others may remain largely unaffected. Returns measured in dollar space are no longer adequate gauges of portfolio performance. Rather, performance is best measured in utility space.

For investors who exhibit a high degree of sensitivity to shifts in wealth (increased risk tolerance when wealth increases and decreased tolerance when wealth declines) a passive or drifting portfolio management strategy may be appropriate. For a two-asset class portfolio (stocks and bonds), when stocks increase in value (i.e. assume a greater proportion of weight in the portfolio), the investor can tolerate the increased risk. However, when stocks decline in value (i.e., the portfolio loses wealth), they constitute a lower percentage of portfolio weight relative to the safer bonds. But this is exactly what the investor may wish. Along the spectrum of possible risk aversion curves, investors exhibiting higher risk aversion as the portfolio approaches a floor value may prefer an insured portfolio strategy; while, at the other end of the spectrum, an investor with low risk aversion may wish to employ a strict contrarian strategy. Between these two extremes lie a series of portfolio management elections that include (1) the tactical asset allocation strategy[75] (appropriate for investors that do not exhibit quite the extreme reaction to wealth changes as the highly risk-averse investor who may prefer portfolio insurance); and, (2) the elections to rebalance towards the strategic asset allocation targets. Rebalance elections are appropriate for investors that exhibit risk aversion curves more in line with the "average" within the population of investors.[76] However, for some investors, it may be prudent to drift![77]

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is risk averse until a wealth target is reached (i.e., when future consumption objectives are fully funded); but, when the goal is attained, the investor becomes more risk seeking with the excess money.

[75] For a review of the historical results of tactical asset allocation vs. fixed mix strategies see, Arshanapalli, B., Coggin, T.D., & Nelson, William, "Is Fixed-Weight Asset Allocation Really Better?" The Journal of Portfolio Management (Spring, 2004), pp. 27-38. The authors compare the asset allocation recommendations of eight major brokerage firms and conclude "when we apply a strict statistical test, none of the eight brokers is significantly different from the [fixed weight allocation]."

[76] For a more complete discussion, see Arnott & Lovell "Rebalancing: Why? When? How Often" Supra, pp. 9-10. See also, Masters, Seth J., Supra: "If an investor is extremely risk-tolerant or, more precisely willing to endure higher risk for higher potential returns, then the benefit of rebalancing is smaller than it would be for an investor who is less risk tolerant." Masters develops a rebalance formula that incorporates a term for costs and a term for investor risk aversion. He concludes that the optimal rebalance strategy is "halfway between the trigger point and the initial target allocation." Dybvig, Philip H., "Mean-Variance Portfolio Rebalancing with Transaction Costs," Working Paper Washington University in Saint Louis (January 2, 2005) points out that Masters' paper computes that non-trading region incorrectly. Dybvig's study examines the shape and location of the no-trade region's boundaries under various types of fixed or variable transaction costs. It is one of a series of studies that apply optimization theory to the question of solving the rebalancing cost/benefit problem. In general, these studies attempt to locate the optimal rebalance point which is defined as the point where the marginal costs of trade execution and tax liability equals the marginal benefit of rebalancing to the targeted asset allocation. See, for example, "The Science and Psychology of Rebalancing, Part 2: Creating the Optimal Approach," (Bernstein Global Wealth Management (2007). Mitchell, John E. & Braun, Stephen, "Rebalancing an Investment Portfolio in the Presence of Transaction Costs," Working Paper Rensselaer Polytechnic Institute (December 16, 2003) develop a transaction cost "efficient frontier" to illustrate the tradeoffs between transaction costs and investment choice. The authors make the important observation that differential transaction costs do not only affect the location of the frontier, but may also reverse buy/sell decisions, and cause significant changes in the weights given to investments in the optimal portfolio. Additional mathematically oriented extensions of this line of research examine the rebalancing cost/benefit question in terms of dynamic

Conclusion

Beneficiaries and fiduciaries would like to know the probable consequences of portfolio management elections prior to their implementation. This is especially the case for decisions to incur voluntarily extra costs and taxes by electing to rebalance the portfolio to its asset allocation targets. A naïve and difficult to defend decision making process is based on data mining historical results to find the "best" combination of asset allocation / rebalancing strategies; where "best" is defined as the combination that, by happenstance, produced the most money or the most favorable Sharpe ratio. Academic research indicates that both utility of wealth and utility of consumption affect asset management policy with respect to both choice of portfolio management approach (drift, fixed mix, insured) and design of rebalance policy. Ultimately, the prudence and suitability of the trust's rebalance policy is a function of its ability to reduce the present value of expected utility loss over the applicable planning horizon. However, given the heterogeneity of trust types, settlor objectives and beneficiary preferences, it is difficult to advocate a "one-size-fits-all" rule for rebalancing especially when such a rule is a product of mere empiricism.

Rebalancing elections have economic consequences; and, therefore, should receive the requisite degree of care, skill and caution by trustees. This is especially the case for taxable portfolios operating under distributional regimes (net income or total return trusts) where cash flow requirements create path dependencies. If, as this essay argues, rebalancing policy is a critical bridge between a trustee's choice of (1) trust portfolio management approach and (2) trust distribution policy, then written investment policy guidelines should make explicit a prudent and suitable rebalancing policy. Indeed, the

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programming algorithms. See, for example, Sun, Walter, Fan, Ayres, Chen, Li-Wei, Schouwenaars, Tom & Albota. Marius A., "Using Dynamic Programming to Optimally Rebalance Portfolios," Journal of Trading (Spring, 2006), pp. 16-27. The dynamic programming approach considers the tradeoff between incurring unconditional rebalancing costs today vs. estimated future rebalancing costs by electing to rebalance partially (or not at all) today conditional on market evolution. Kritzman, Mark, Myrgren, Simon & Page, Sebastien, "Portfolio Rebalancing: A Test of the Markowitz-van Dijk Heuristic," Working Paper MIT Sloan School of Management (March, 2007), compare the dynamic programming approach to a more tractable quadratic function for investors with log utility of wealth owning various sized portfolios.

[77] A similar conclusion was reached in a study that compared the utility value of rebalance strategies to investors with dissimilar utility of wealth functions. See, Clark, Truman A., "Efficient Portfolio Rebalancing," www.dfafunds.com (Fall, 2001). Bernstein, William J., "The Rebalancing Bonus: Theory and Practice," Efficient Frontier (1996) Supra, presents another possible answer to the question "when is it prudent to drift?" Bernstein, entertaining a Capital Asset Pricing Model approach to asset management, assumes a well-diversified portfolio of indexed products replicating the world-market of tradable financial assets. As capital markets earn differing returns, rebalancing is unnecessary because the capitalization weighted market portfolio should remain efficient. This observation, however, is true only for a 100% risky asset portfolio; and does not consider whether the market portfolio is well suited to the purposes, terms, distribution requirements, and other circumstances of the trust. Trustees are rarely charged with a duty to replicate a paper portfolio, to beat the market, or any such other investment goal. Rather, they are charged with managing the portfolio so that its future evolution matches settlor objectives and beneficiary utility. Furthermore, as a portfolio drifts, there is concomitant drift in the magnitude of factor risk exposures. Simplistically, risk factors (inflation sensitivity as measured by bond duration and convexity or by equity duration metrics, default risk as measured by bond credit risk and equity bankruptcy risk, etc.), which may have been carefully calibrated to the trust portfolio's economic objectives, may deviate from prudent bounds.

trustee may find that the rebalance strategy optimal for a net income trust distribution regime is not optimal for a total return unitrust distribution regime, or a total return indexed annuity distribution regime.

The right strategy depends primarily on investor utility. This is especially true for asset decumulation regimes. If unspent terminal wealth (final portfolio value) has value because of bequest preferences or remainder interest considerations, then the trustee will select distribution policies and rebalance strategies that will augment the utility of final dollar values. If however, the settlor does not have these preferences, the unspent money may merely represent lost consumption opportunities for the current beneficiary and may produce disutility. In this case, asset management and rebalance strategies may give way to actuarial solutions such as immediate annuities. This is a complex extension of this paper, however, and, given the current transaction-biased and commission-oriented marketplace--which is replete with potential counterparty risk--for actuarial instruments, is itself a solution that may bear considerable risk.

Rebalance strategies appropriate for wealth accumulation trusts may not be appropriate for decumulation trusts. At the limit, for trusts operating under long-term inter-generational planning horizons, rebalancing may not be preferred. In such cases, the rebalancing policy may devolve into a no-rebalance policy. In other cases, the rebalance policy must not only be appropriate to the purposes, terms, distribution requirements, and other circumstances of the trust, but must also match the skill sets of the trustee or the money manager to whom the trustee delegates investment responsibilities. The high dimensionality of the problem argues for simulation analysis so that the trustee can “test drive” various asset management approaches (including asset allocations) and rebalance elections within the distributional constraints and limits of discretion established in the governing instrument.