CHAPTER 7: Portfolio Management & Investor Risk Tolerance

Changes in personal wealth can alter an investor's risk tolerance.[1] If a portfolio increases in value, some investors tolerate risk better because they have a cushion against possible future downturns, while others become more conservative because increased wealth can enable them to reach their goals with less portfolio risk. But changes in risk tolerance are highly personal, and each investor has a unique sensitivity to changes in wealth.

A fully integrated portfolio management system responds to changes in wealth and risk tolerance.

1. It enhances investor utility by creating an initial asset mix at an appropriate level of risk and reward; and,
2. It monitors changes in investor circumstances to assure that the portfolio continues to be aligned with the investor's evolving purposes, distribution requirements, and financial circumstances – including a changing predilection for investment risk.

This chapter advances earlier discussions of asset allocation by considering relationships among three elements: personal risk tolerance, asset allocation strategies, and portfolio management approaches.

ASSET ALLOCATION STRATEGIES

As a practical matter, asset allocation can follow one of several strategies. By 'strategy' we mean portfolio management that falls into one of three categories:

- Market anticipating (tactical asset allocation)
- Market agnostic (strategic asset allocation, whether buy-and-hold, or constant mix)
- Market reactive (floor + multiplier insured portfolio strategy)[2]

Tactical Asset Allocation

Tactical asset allocation assumes that the investor's risk aversion is fixed. Changes in wealth do not therefore affect long-term strategic allocations. However, asset price changes do influence short- and medium-term capital market expectations (expected returns, risks, and market correlations). Therefore, the portfolio manager is willing to deviate temporarily from the long-term allocation if a market forecast indicates attractive buying or selling opportunities. Several well-known portfolio management approaches flow from this viewpoint. Market forecasting is the basis for a sector rotation approach – overweighting the

[1] Riley, William B. & Chow. K. Victor, "Asset Allocation and Individual Risk Aversion," Financial Analysts Journal (November/December, 1992), pp. 32-37.

[2] The chapter provides further clarification of these terms.

sectors of the market that are expected to produce relatively superior performance, while underweighting less promising sectors. At the extremes, tactical asset allocation leads either to strict contrarian approaches, or to market timing systems. In a tactical asset allocation approach, the portfolio manager represents that he can anticipate the direction and timing of price changes, and tries to capitalize on this forecasting ability by positioning the portfolio to take advantage of his investment predictions.

Strategic Asset Allocation

Strategic asset allocation determines the investor's desired exposure to sources of systematic risk. It is premised on the belief that capital markets are generally efficient. As noted in Chapter One, an efficient market rapidly incorporates all known information regarding economic and financial matters into the current price of a security. Thus any current asset price represents the consensus opinion of the asset's value. Price changes occur because:

- New information causes a reassessment of the asset's value; or
- There are more liquidity sellers than information buyers (but this results from random chance; no investor has the inside track on the direction of price change).

Under this theory, the prices of securities are exactly what the market requires to clear. Once the asset allocation mix is set, the portfolio remains fixed for relatively long periods of time. Only significant changes in an investor's personal circumstances or risk tolerance prompt reassessment of portfolio allocation. Among the asset management approaches that flow from this viewpoint are Buy-and-Hold and Constant Mix styles. A Buy-and-Hold investor, after setting an initial asset allocation, takes no further actions to preserve the target weights of portfolio asset classes. A Constant Mix investor periodically rebalances the portfolio to its initial asset allocation by selling positions in assets with recent high returns to finance additional purchases of assets with recent low returns.

INSURED ASSET ALLOCATION: THE FLOOR + MULTIPLIER STRATEGY

Insured Asset Allocation also takes the efficient market viewpoint that changes in price lack predictability sufficient to form profitable market-anticipating /market-beating systems. However, it assumes that changes in wealth influence investor risk aversion. As portfolio value declines, the investor becomes more sensitive to investment risks; as the value increases, the investor becomes more comfortable with risk. A common method for implementing the Insured Asset Allocation strategy is through a Floor + Multiplier approach to portfolio management.

The approach sets a floor on the dollar value of a portfolio. As the portfolio value approaches the floor, assets are shifted to risk-free Treasuries. If the value sinks to the floor value, the entire portfolio will consist of Treasury Bills, and will suffer no further declines. Conversely, as portfolio value climbs above the floor, more funds are committed to equities. Many Floor + Multiplier portfolios have equity commitments of two to four times the spread between current portfolio value and the floor value. This difference is called a "multiplier."[3]

FIGURE 7-1 summarizes this discussion:

- Asset Allocation Strategies
- Tactical Strategic Insured
- Market Anticipating Market Agnostic Market Reactive
- Market Timing/Sector Rotation Buy-and-Hold/ Contant Mix Floor + Multiplier

[3] For a detailed discussion of portfolio allocation strategies for differing investor risk preferences see, Collins, Patrick J. and Stampfli, Josh, "Managing Private Wealth: Matching Investment Policy to Investor Risk Preferences," The Banking Law Journal (November/December, 2009), pp. 923-958. This is available on the Schultz Collins website.

Therefore, on a preliminary basis, if an investor believes that a money manager can profitably anticipate market events, all else equal, the prudent course of action might be to initiate and closely monitor a tactical asset allocation strategy for ongoing portfolio management. If an investor believes that it is too risky, costly or impractical to try to out-predict the market, all else equal, the prudent course of action might be to initiate and closely monitor either a strategic asset allocation strategy – Buy-and Hold or Constant Mix management approach – or, an Insured Asset Allocation strategy – the floor + multiplier approach.

The payoff to portfolio management approaches based on a tactical asset allocation strategy are primarily a function of manager skill. Sophisticated statistical testing and monitoring is required to distinguish skill from luck; or, as one article puts it, to distinguish between the skilled monkey and the unlucky manager.[4] Rather than pursuing a complicated discussion regarding track record analysis and inferential statistics, the remainder of this chapter therefore focuses on the payoffs to asset management approaches that do not employ tactical asset allocation.

The issue under examination is which payoff structure best aligns with investor risk tolerance.

Theoretical Payoffs to Different Asset Management Approaches

FIGURE 7-1 depicts the value of a hypothetical portfolio (on the Y-axis) under three asset management approaches as the value of the risky assets (on the X-axis) changes. We consider the Buy and Hold, the Constant Mix and the Floor + Multiplier approaches.[5] The Buy and Hold portfolio assumes an initial commitment of 60% risky assets to 40% T-Bills. The Constant Mix assumes constantly rebalancing to 60% equity/40% T-Bill. The Insured Portfolio approach assumes a floor value of 70 with a multiplier of two. Therefore, the initial equity investment position of the Insured Portfolio is (100-70) x 2 = 60% equity/40% T-Bill. Although each portfolio starts with the same ratio of equity to risk-free asset, each diverges in value as the price of the risky asset portion changes.

COMPARATIVE PERFORMANCE OF ASSET MANAGEMENT STYLES

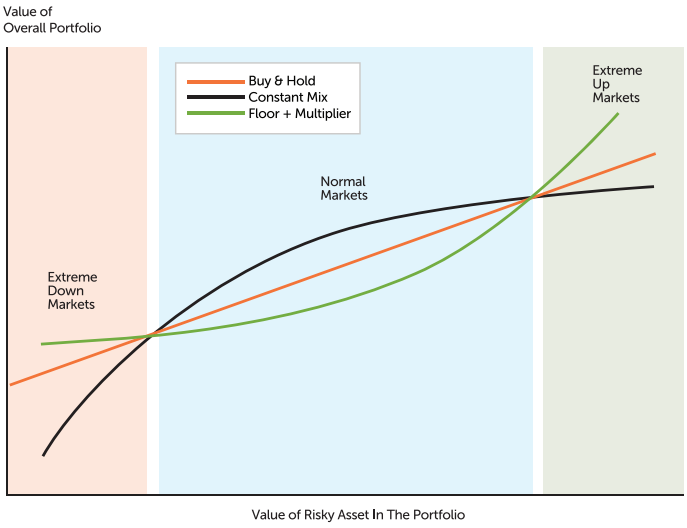

FIGURE 7-1

Buy and Hold

In the case illustrated above, the Buy and Hold investor placed 40% of the portfolio in short-term T-bills and 60% in the risky market portfolio (a combination of stocks, bonds and real estate). What are the implications?

- The value of the portfolio will not fall below

[4] Vermorken, Maximilian, Gendebien, Marc, Vermorken, Alphons and Schroder, Thomas, "Skilled Monkey or Unlucky Manager?" Journal of Asset Management (October, 2013), pp. 267-277.

[5] We note that this example is highly stylized. Although there are many factors influencing investment decisions, this discussion abstracts away from many of them in order to focus more tightly on modeling the theoretical payoffs to each approach. In Chapter Eight, there is a discussion of implementing and maintaining specific portfolio management approaches in the face of trading costs and other portfolio frictions.

that of the 40% commitment to T-Bills;
- The portfolio has unlimited upside potential;
- The future value of the portfolio is (approximately) linearly related to the performance of the risky asset portion, with the rate (slope of the line) of future value change equal to the proportion of the portfolio committed to the risky assets (in this case, the initial commitment is 0.60).

Constant Mix

Investors whose risk aversion is not greatly affected by changes in wealth will employ a buy low/sell high strategy. If an asset price declines, they will buy into the falling market. Conversely, they will take profits by selling into rising markets. A typical example of this strategy is the Constant Mix management style. The Constant Mix strategy restores the asset allocation to its original proportion of risk-free and risky assets as market prices change. What are the implications?

- Buying into declining markets while readjusting the equity portion to a constant percentage of portfolio value means that, theoretically, 100% of the portfolio is exposed to risk;
- As markets increase in value, equities are sold. Therefore, in a trending 'up' market, the investment payoff will tend to lag behind the payoff for a buy-and-hold portfolio that does not trim back equities;
- As equity markets decrease in value, low risk assets are sold and equities are purchased to maintain targeted allocation levels. Therefore, in a trending 'down' market, returns will also lag those of a buy-and-hold portfolio;
- The future change in portfolio value has a concave (turned down curve) slope, which lies above the straight-line buy-and-hold payoff line in many market conditions; but lies below it in strongly trending markets.

Insured Asset Allocation

Investors whose risk aversion exhibits greater than average sensitivity to changes in wealth may employ a buy high/sell low strategy. This is a momentum driven strategy best characterized by the floor + multiplier portfolio management style. What are the implications?

- The portfolio is only exposed to risk on the amount above the floor value;
- The rate of future change in portfolio value depends on the percentage of the commitment to risky assets when the dollar value is above the floor value. On a $1,000,000 portfolio, for example, with a floor value of $800,000 and an risky asset multiplier of 3, the equity commitment equals (1,000,000-800,000) x 3 = $600,000;
- As markets rise, the commitment to risky assets means that the portfolio increases its exposure to equities, creating a positive feedback loop with beneficial effects on overall return;
- As markets fall, the commitment to risky assets scales back (by a factor of three in the above example), creating a beneficial negative feedback loop until the value of the portfolio reaches its insured value or floor;
- The investment payoff will be convex (a turned up curve) and will outperform both the buy-and-hold straight-line payoff and the Constant Mix allocation payoff in extreme up or down market conditions.

The ability of the floor + multiplier approach to protect investors on the downside while producing attractive returns in a strong bull market is alluring to many investors. In the mid-1980's, more professionally managed money used variations of the insured portfolio system (buy high/sell low) than the Constant Mix (buy low/sell high) system. By October 1987, when

market prices started to drop, approximately 20% of institutional investors wanted to buy while 80% wanted to sell. Many economists believe this order imbalance was a primary contributor to the 1987 market crash. During the crash, severe price declines eliminated the entire buy side of the market. As a result, insured portfolios could not sell, and, in some cases, dropped far below their targeted floor values. Thus, in a market crash, when portfolio insurance was needed most, it was ineffective.

...there is no "best" asset management approach, because the payoff to each approach differs depending on market conditions.

If either asset management approach dominates the money management community, it sows the seeds of its own destruction. Insured portfolio management creates market volatility, because its momentum driven trading exacerbates market swings. However, in volatile markets, it may not provide the investment payoff functions it promises. Conversely, Constant Mix portfolio management reduces market volatility by buying when prices are falling and selling as prices rise. But Constant Mix strategies require volatility (sudden price reversals) to make contrarian bets worthwhile. Hence, one may reasonably conclude there is no "best" asset management approach, because the payoff to each approach differs depending on market conditions.

The shape of the payoff curve for each strategy depends on equity market volatility. In trending markets, the Floor + Multiplier approach buys into the favorable trend and rides the winners up. The more the winners win, the better the portfolio's performance. Conversely, the Constant Mix portfolio management approach pares back the winners (takes profits on the way up) to maintain the proportionate value of the fixed income anchor position. Therefore, when markets trend up or down over long periods, this portfolio management approach has relatively poor performance when compared to either the Floor + Multiplier or Buy-and-Hold styles. When market trends reverse, or are nonexistent, however, these results also reverse. The Constant Mix portfolio outperforms other approaches by taking advantage of asset price changes.[6]

ASSET MANAGEMENT APPROACHES AND INVESTOR RISK AVERSION: A KEY TO INVESTMENT POLICY

In the investment text Managing Investment Portfolios: A Dynamic Process, the authors state: "the appropriateness of buy-and-hold, constant mix, and constant-proportion portfolio insurance strategies for an investor depends on the investor's risk tolerance, the types of risk with which she is concerned (e.g., floor values or downside risk), and asset-class return expectations."[7]

Briefly, investors whose risk aversion exhibits significantly greater than average sensitivity to changes in wealth will employ a buy high/sell low strategy. This suggests a Floor + Multiplier portfolio management approach. Investors with average risk aversion will employ a buy low/sell high strategy. If an asset

[6] A more advanced discussion of these asset management approaches is found in Collins, Patrick J., and Stampfli, Josh, "Managing Private Wealth: Matching Investment Policy to Investor Risk Preferences," The Banking Law Journal (November/December, 2009), pp. 923-958. This is available on the Schultz Collins website.

[7] Chapter 13 "Monitoring and Rebalancing," Maginn, John L., Tuttle, Donald L., Pinto, Jerald E., and McLeavey, Dennis W., eds. Managing Investment Portfolios: A Dynamic Process (The CFA Institute, 2007), p. 714.

price declines, they will buy into the falling market. Conversely, they will take profits by selling into rising markets. This suggests that such an investor is best served by a Constant Mix management style. Some investors exhibit decreasing risk aversion as wealth increases/increasing risk aversion as wealth decreases. They may prefer a Buy-and-Hold approach.

Along the spectrum of possible risk aversion, highly risk-averse investors may prefer a portfolio insurance approach. At the other end of the spectrum, an investor with low risk aversion may prefer a strict contrarian approach. Between these two extremes lie a series of portfolio management elections that include elections to rebalance towards the strategic asset allocation targets. Rebalance elections are appropriate for investors that exhibit risk aversion more in line with the "average" within the population of investors.

The important point to note is that some investors are highly affected by shifts in wealth while others may remain largely unaffected. Returns measured in dollars are no longer adequate gauges of portfolio performance. Rather, performance is often best measured in utility space. The following sections elaborate on this topic.

THREE CASE STUDIES

Assume for discussion purposes that a portfolio holds stocks primarily for "growth;" and holds bonds primarily for "safety." Simplistically, Stocks = Growth; Bonds = Safety. We offer case studies based on three investor risk profiles.

Risk Profile One: Investor Not Sensitive to Changes in Portfolio Value

An investor seeks advice on how to invest a portfolio to support retirement goals. After consultation with an investment advisor, including a review of historical returns generated by various stock and bond combinations, the investor decides that he is comfortable with an allocation of 60% stock/40% bond. The advisor informs him that such a portfolio exhibits a maximum yearly downside risk of approximately 30% (as measured by two standard deviations below its historical average return), but it is possible that such a portfolio could have a greater fall in value. The advisor creates an Investment Policy Statement [IPS] that memorializes future portfolio management guidelines. The IPS indicates that the portfolio will adhere closely (±10%) to the 60-40 asset allocation target.

The advisor informs the client that such an asset management approach is "disciplined." This means that if stocks go up in value, the portfolio will continue to adhere to its strategic asset allocation target by selling growth and buying safety. Sales will occur when stocks are up in value and this is good because it imposes a 'sell high' discipline. If, however, stocks decline in value, the portfolio will sell safety and buy growth. This is also represented as a good thing because it imposes a 'buy low' investment discipline. The advisor's educational program argues that, under a disciplined investment approach, maintaining a lower than target equity allocation is sub-optimal and selling all stocks during a bear market is a bad thing – because it amounts to timing the market – a bootless endeavor. Contemplating such a step suggests that the client may be acting irrationally by allowing emotions to dictate investment strategy.

Therefore, the investment discipline is:

1. Sell high/Buy low
2. Maintain the initial growth/safety ratio throughout all market conditions
3. A variant of a 'strict contrarian' investment strategy – sell into growth and buy into market declines.

FOCUS ON BEAR MARKET

- In a prolonged bear market in stocks the

investor sells safety to buy risk – i.e., he sells his bonds to buy stock.
- The risk to client wealth therefore increases at an increasing rate. Not only does the value of equity continue to decline during the bear market, but the investor is eliminating safety at just the time when it is most needed.
- If cash is removed from the portfolio for retirement income during the bear market in stocks, the investor accelerates the decline in wealth.
- Risk goes into the red zone if the bear market is long or severe.
- Discipline may mean the investor is first on his block to visit the soup kitchen.

SUITABILITY

This risk management approach is appropriate for investors who:

- Are not concerned about increasing losses in a bear market.
- Have investment goals that are not critically important – the goal is something they would like to do rather than something they must do.
- Have high wealth to consumption ratios (lots of wealth but little need for substantial cash flow from the portfolio).
- Have long-term planning horizons;
- Have asset accumulation objectives – as opposed to spending objectives;
- Are still making periodic contributions towards funding a long-term goal – i.e., investors with labor income rather than retirees.

GENERAL RESULTS

Under this risk management approach, the investor is willing to sacrifice performance in extreme bull and bear markets in order to enhance performance in an average market. "On average," the client will do well in such a risk management system. But an investor owns only a single portfolio and has only one chance to assure that it is sufficient to fund critical needs. The investor must live with the actual portfolio rather than with the cold comfort that, on average, investors do well.

This portfolio management approach offers a concave payoff function – good during normal markets but worse during market extremes. It is not appropriate for clients who are sensitive to changes in their current wealth. A risk management approach that accelerates the decline of wealth to the point where a portfolio can no longer support the desired objective(s) is imprudent. Staying the course in the hope that someday ("in the long run") the growth part of the portfolio will rebound and the lost dollars will return, is not investment discipline – it is mere hope.

An investor must assess both his willingness to take risk and his ability to endure it. If the investor is acting as a fiduciary – investing funds for the benefit of others – it may be a breach of fiduciary duty to recommend and facilitate a course of action that will substantially increase the probability that critical investment goals become infeasible.

CAVEAT

Portfolio management presents the investor with a series of asset management options. However, the highly stylized case studies in this section ignore flexibility in deciding whether to exercise rebalancing options. For example, it is unlikely that an investor would be so myopic that he would fail to recognize that, in a severe and prolonged bear market, he has the option to refrain from rebalancing the portfolio, thus tilting towards risky assets. Portfolio management guidelines, as codified in a written Investment Policy Statement, are not carved in stone. Rather, they act as asset management guidelines that remind the investor that short-term actions require careful thought lest they undercut long-term objectives.

Risk Profile Two: Investor Moderately Sensitive to Changes in Current Wealth

Assume, in this risk profile, the same fact pattern. After reviewing historical data, the investor tells the advisor he is comfortable with a 60/40 allocation. However, he tells the advisor that if his wealth increases – the portfolio's growth element (stocks) goes up in value – he is OK with maintaining the increased exposure to equity risk. After all, at that point he is "playing with house money." If portfolio growth turns negative, however, he does not want to sell safety during a bear market.

The advisor understands that the client wants a fixed "safety" component to protect against the downside and a variable growth component for the upside. This means that only changes in stock prices will drive a change in investor wealth.

The advisor memorializes the clients risk preferences and constraints in an Investment Policy Statement calling for a static bond position – constant safety and variable growth position. This is a Buy-and-Hold Investment approach to portfolio risk management.[8]

FOCUS ON BEAR MARKET

Portfolio risk – the risk of failing to achieve a target return or a dollar wealth goal *increases at a decreasing rate*: stocks continue to decline in value, but the rate of overall wealth decrease slows because equity constitutes an ever smaller portion of the aggregate portfolio.

SUITABILITY

This risk management approach is appropriate for investors who:

- Have moderately important goals or shorter planning horizons.
- Have a moderate wealth to consumption ratio – there is a low probability that demands for cash will deplete the portfolio.

The Investor is willing to take higher equity risks in a bull market environment. In a bull market, portfolio risk increases at an increasing rate because the proportion of growth to safety increases. However, the opposite is true during bear market environments.

The payoff function from this risk management approach is approximately linear – wealth changes at the rate of change determined by the current value of equity in the portfolio.

Risk Profile Three: Investor Extremely Sensitive to Changes in Current Portfolio Value

Assume, in this risk profile, the same fact pattern. After reviewing historical data, the client tells the advisor he is comfortable with a 60/40 allocation. However, the client states that if his wealth increases – the portfolio's growth element (stocks) goes up in value – he is willing to increase risk at an increasing rate. Such a client might be willing to margin a portfolio (or hold leveraged ETFs) to capture as much bull market return as possible. However, if wealth decreases, the probability of a shortfall relative to the client's goal increases. Therefore, the client is not willing to incur declines below the point at which portfolio goals cease to remain feasible.

The advisor memorializes the clients risk preferences and constraints in an Investment Policy Statement calling for a dynamic risk-controlled asset management approach. As the growth element increases in value, the client has a greater margin of safety and, therefore, is willing to take more risk.

[8] Please note that under a Buy-and-Hold investment management approach the stock and bond components can be internally rebalanced – selling stocks to buy bonds (or vice versa), however, is not contemplated.

However, as wealth declines towards a critical "feasibility" boundary, the equity positions are unwound. By the time the portfolio reaches the critical boundary, equity has been eliminated and only safety remains. This is an example of a floor + multiplier portfolio management approach.

FOCUS ON BEAR MARKET

- Under this portfolio management approach, reducing equity investments is a good thing – not an irrational response based on fear.
- As the bear market unfolds and wealth approaches the investor's stop-loss limit, equity positions are systematically unwound. An equity position is maintained only above the minimum value required to fund critical goals.
- Because it is dynamic rather than static, asset allocation is always calibrated to the investor's risk preferences and constraints.

SUITABILITY

This risk management approach is appropriate for investors who:

- Have a low wealth/consumption ratio – especially in portfolio distribution mode.
- Have critical planning objectives – things they must do rather than things they would like to do.
- Lack labor income – e.g., retirees.
- Investors with strong "state preference" utility of wealth functions.[9]

▲ PRUDENT INVESTMENT POLICY:

INVESTOR RISK PREFERENCES & CONSTRAINTS

This section considers portfolio design and asset management election issues in a somewhat more technical light. It offers a short discussion of how an investor's attitudes towards risk – i.e., the investor's risk tolerance function – influences investment policy. Initially, this book asked the reader to think about the implications of the following statement: "more money is better than less." Financial economists use the term "utility" to express the satisfaction of adding a dollar to wealth; and, not surprisingly, use the term 'disutility' to describe the pain of subtracting a dollar from wealth. Each investor has a risk limit beyond which he becomes uncomfortable. If the only way to add additional dollars to a portfolio is to pursue a strategy that prevents a good night's sleep, then the satisfaction ("utility") of the expected financial reward is negated by the discomfort ("disutility") of violating an acceptable constraint on risk. Here is the critical point: the risk/reward tradeoff (discussed in Chapter Five) must be translated into an equivalent tradeoff that accounts for the preferences and constraints of each investor. Specifically, the investor translates the risk/reward tradeoff into the desired return/sleep tight tradeoff. When the tradeoff ratios are exactly in sync, the investor has found the portfolio that produces the greatest utility.

There is a general correspondence between the risk/reward tradeoffs available in the capital markets and the desired return/sleep tight tradeoff that each investor prefers.[10] Likewise, there are direct mathematical relationships among "utility" – a measure of how satisfaction changes with the addition or subtraction of wealth – "risk tolerance" – a measure of how the

[9] We explain State Preference utility functions below.

[10] Sometimes this is expressed as a fear/greed tradeoff. Although this book does not explicitly discuss theories from Behavioral Finance, the decision making process faced by individual investors forms the subject of experiments in investment decision making. Behavioral Finance, although criticized for its lack of sound theoretical underpinning, nevertheless offers both interesting observations and helpful vocabulary for describing investor predispositions. Interested readers may consult the essay entitled "The Great Debate: Behavioral vs. Standard Finance: Are Investors Rational?" from the ***Investment Quarterly 2001 Q1***. This is available on the Schultz Collins website.

Fortunately, an investor does not have to master mathematics in order to form a prudent and suitable investment program. This said, an investor benefits from understanding something about these concepts lest a persuasive "story" spun out by a product or service vendor obscures or overcomes the principles of investment prudence.

rate of satisfaction changes at various wealth levels[11] – and, "risk aversion" – a measure of how an investor's aversion to uncertainty changes at various wealth levels.[12]

Fortunately, an investor does not have to master mathematics in order to form a prudent and suitable investment program. This said, an investor benefits from understanding something about these concepts lest a persuasive "story" spun out by a product or service vendor obscures or overcomes the principles of investment prudence. The following section, therefore, is a brief introduction to 'risk aversion' – the flip side of risk tolerance. A highly risk averse investor exhibits little tolerance for investment risks.

MATCHING THE PORTFOLIO TO INVESTOR RISK PREFERENCES AND CONSTRAINTS

Risk Aversion

Given the complexity of the topic, this section requires a more technical narrative. Risk aversion sets acceptable bounds for portfolio risk. The word 'bounds' is plural because, as discussed in Chapter One, there are several types of investment risk and various ways to measure them.[13]

Here is a list of commonly found risk aversion functions:

- Investors exhibiting Constant Absolute Risk Aversion [CARA] will not risk more than a specific dollar amount on an uncertain venture – "throughout the planning horizon, only $X at risk in the stock market – not a penny less; not a penny more";
- Investors exhibiting Constant Relative Risk Aversion [CRRA] will not risk more than a specific fraction of their wealth on an uncertain venture – "let's keep a constant 70% of my wealth exposed to the risks and

[11] Risk tolerance is the first derivative of utility.

[12] Risk aversion is the reciprocal of risk tolerance. Mapping risk aversion over the entire range of investment wealth instead of just one or a few wealth levels is performed using a risk aversion function. Risk aversion curves are also known as "indifference curves." An indifference curve plots the series of increasingly risky investments that, as a result of their higher expected returns, all provide equal utility to the investor (hence, the term "indifference"). The more sensitive the investor to a change in wealth, the steeper the curvature – that is to say, the greater the risk premium required to induce the investor away from the risk-free rate. The steepness of the risk aversion curve is mathematically equivalent to the change ("elasticity") of marginal utility at any given wealth level. Given that the utility of wealth curves are generally upward slopping – at a decreasing rate of acceleration as wealth grows larger – it follows mathematically that the upwardly sloping curves have a positive "velocity" and a negative "acceleration." For readers familiar with calculus, the curves have a positive first derivative and a negative second derivative. Although each investor has a unique attitude towards risk [and, therefore, different preferred risk/reward tradeoffs], it is generally true that the risk aversion function is expressed as follows:

Risk Aversion =-(second derivative of the utility of wealth)/(first derivative of the utility of wealth).

In other words, an investor's risk aversion function can be derived from the shape of his utility of wealth curve; and utility of wealth can be recovered from his risk aversion curve. The third derivative of an investor's utility function is known as "prudence." It forms the motivation for precautionary savings. For further discussion, see Collins, Patrick J., "Managing Retirement Portfolio Withdrawals in Turbulent Times: Precautionary Savings, Investment Reserves, and Mid-Term Adjustments". This is available on the Schultz Collins website.

[13] Risk encompasses statistical metrics such as 'standard deviation,' 'range,' and 'variance;' downside metrics such as 'shortfall probability,' 'shortfall magnitude,' 'risk to investment principal;' and factor risks such as 'market risk,' value' risk, etc. It is worth restating that characterizing an investment policy as "low risk," or "high risk" is not helpful in terms of setting the investment policy guidelines for portfolio management. Behavioral finance often characterizes "risk" aversion as "loss" aversion.

Event	Risk Aversion Function	Preferred Wealth Management Response	Example
Increase in Value	CARA	Preserve the Gain	Investor sells the gains in risky assets and puts profits into risk-free investments.
	CRRA	Rebalance to Target Allocation ("stay the course")	Investor sells risky assets to maintain asset allocation – i.e., rebalance to target asset allocation. This is a Constant Mix management approach.
	DARA	Add more to the winners	Investor increases commitment to risky assets in excess of original dollar amount or targeted allocation percentage. This is a Floor + Multiplier Strategy.
	DRRA	Let it ride – I have a "cushion"	Investor maintains a Buy-and-Hold Strategy.
Decrease in Value	CARA	Limit absolute amount at risk	Investor holds risky assets only up to the initial dollar value at risk. No additional money goes towards the purchase of risky assets.
	CRRA	Rebalance to Target Allocation ("stay the course")	Investor buys risky assets to maintain target asset allocation – i.e., rebalance to target asset allocation. This is a Constant Mix management approach.
	DARA	Sell Growth and Buy Safety	Investor sells risky assets to reflect the fact that decreased wealth leads to decreased risk tolerance. This is a Floor + Multiplier Strategy.
	DRRA	Do not "feed the bear"	Investor maintains a Buy-and-Hold Strategy.

FIGURE 7-2

rewards of stocks";

- Investors exhibiting Decreasing Absolute Risk Aversion [DARA] will risk a greater dollar value of wealth as wealth increases – "If stock prices are increasing, let's add more money";
- Investors exhibiting Decreasing Relative Risk Aversion [DRRA] will risk a greater fraction of wealth as the dollar value of wealth increases – "If stock prices are increasing, it is OK to let my fractional allocation to risky assets increase proportionately."

FIGURE 7-2 summarizes common investor reactions to changes in portfolio value.

Differing risk aversion functions lead to differing portfolio management preferences. For example, the CRRA risk aversion function generally encourages investors to stay the course in both up and down markets. One striking characteristic of the CRRA function is that it is independent of wealth. That is to say, the CRRA investor maintains the same asset allocation irrespective of changes in portfolio value. This seems strange; and it is difficult to imagine that many investors would be willing to adopt such a portfolio management approach. However, it is the most common form of advice – "stay the course," "maintain discipline," "do not try to outguess or time the market." Why is this so?

In most market conditions, the Constant Mix portfolio management approach produces a higher return than the return produced through alternate

More than any other topic in finance, state preference utility offers a fascinating and challenging counterpoint to the conventional wisdom surrounding investment decision making.

approaches such as Buy-and-Hold or Floor + Multiplier.[14] Generally, it is during extreme bull or bear market conditions that the Constant Mix approach fails to deliver relative outperformance. Furthermore, the trading and portfolio management tasks associated with the Constant Mix approach are well within the ability of most investment advisory firms to manage.[15] It is an asset management approach that is well-suited to a portfolio in the wealth accumulation stage of the investment life cycle.[16] Finally, it provides an ongoing election regarding periodic rebalancing. The election not to exercise the option to rebalance means that, during particularly distressing market conditions, the investor is not forced to jettison lower risk assets.

We also point out that:

- The Floor + Multiplier approach requires liquid markets so that leveraged equity positions – the multiplier – can be unwound at a reasonable cost and within a reasonable time. However, bear markets are characterized by liquidity shortages as investors pile up demand-to-sell pressure to the point where it may overwhelm demand-to-buy. Such market conditions produce price discontinuities that create a positive probability that the minimum floor guarantee cannot be assured.
- The Buy-and-Hold approach, on the other hand, requires a substantial initial commitment to the risk-free asset in order to establish a meaningful "floor" for the portfolio. The opportunity cost of such an approach often makes it unattractive to many investors.
- Other than the all T-Bill portfolio (an approach that has far-reaching opportunity costs when measured by its long-term expected dollar value), there is considerable downside risk in each asset management approach.

STATE PREFERENCE UTILITY

More than any other topic in finance, state preference utility offers a fascinating and challenging counterpoint to the conventional wisdom surrounding investment decision making. This section uses state preference utility to reconsider the proposition that the proper goal of a portfolio is to maximize return.

Assume a future economy that has only five states of the world. In this economy the portfolio can hold risk-free investments or can invest in risky investments. The risk-free rate of return is 2%. The investor forms beliefs concerning the probability of the occurrence of each state and the payoff per dollar of initial portfolio wealth in each state. Payoffs represent consumption opportunities – e.g., retirement income – available in each state. **FIGURE 7-3** summarizes the investor's beliefs.

The expected return overall economic states [6%] is greater than the risk-free return [2%]. Therefore, the

[14] Refer **Figure 7-1** at the beginning of this chapter: "Comparative Performance of Investment Management Styles." Most investors have little patience with sub-par performance; and, many gravitate towards the approach that provides, on average, the greatest chance of attractive returns.

[15] The dark side of this statement is that there are investment advisory firms who believe that setting an initial asset allocation constitutes an effective and thorough risk management program. Sometimes the mantra of "stay the course," devolves into a belief that the advisor need not use ongoing care, skill and caution in portfolio management.

[16] Much academic research employs mathematical models calibrated to the investor life cycle. This term generally refers to a multi-stage planning horizon during which the investor saves and invests during years with labor income surplus and draws down financial resources during retirement. The pre-retirement period is an accumulation phase; the post-retirement period is a distribution phase.

Economic State	Probability of Invested	Payoff per $1	Expected Return [Probability x Payoff]
Depression	5%	$0.40	$0.02
Recession	15%	$0.80	$0.12
Normal	60%	$1.10	$0.66
Prosperity	15%	$1.20	$0.18
Boom	5%	$1.60	$0.08
Expected Return (Sum of Probability-Adjusted Payoffs - 1)		6%	

FIGURE 7-3

commonly employed decision rule is to select the risky asset portfolio (more money is better than less).

If, however, the investor does not have an equal preference for returns across all possible economic states, there is a need for a different decision rule. For example, an investor may value returns received in contraction states more than returns received in growth states – 'enough to eat' vs. 'keeping up with the Joneses.' **FIGURE 7-4** summarizes the investor's preferences.

The column labeled 'Subjective Discount Factor' is new. It indicates that the investor values $1.00 at its full face value during an economic depression. Dollars are hard to come by in poor economies, and are fully valued for the consumption opportunities they offer. However, when dollars are plentiful and easily obtained during prosperous economies, each dollar carries a lower valuation – at least subjectively. It is easier to spend a dollar when you have a lot of them.

Under state preference utility, there is a different decision rule. Here, the subjectively-adjusted return over all economic states [1%] is less than the current risk-free return [2%].[17] Therefore, the investor does not elect to invest in the portfolio that offers the higher expected return. He elects to remain in the risk-free asset because the fear of experiencing a low-consumption state outcome is greater than the prospect of attractive consumption opportunities during a prosperous state. A state preference approach to asset management decision making often differs from a more traditional maximization of utility over all economic states approach.[18]

Economic State	Probability of Economic State	Payoff	Subjective Discount Factor	Discounted Expected Return [Probability x Payoff x Discount Factor]
Depression	5%	$0.40	1.00	$0.02
Recession	15%	$0.80	0.99	$0.12
Normal	60%	$1.10	0.98	$0.65
Prosperity	15%	$1.20	0.90	$0.16
Boom	5%	$1.60	0.80	$0.06
State Preference Value of Portfolio Return				1%

FIGURE 7-4

[17] The sum of the returns in the far right column amounts to $1.01, or, a one-percent rate of return on the original $1.00 investment.

[18] The concept of "time preference" is comparable to "state preference" in the administration of retirement income portfolios. Retirees may place greater value on consumption during the early years of retirement. Foregoing consumption opportunities today in order to assure that future funds are on hand given a lower-probability extended life span is, for some investors, not an attractive proposition. Such investors may wish to increase utility by "front loading" retirement spending. When threshold expenses must be funded in every period, a retiree is said to exhibit an "inelastic intertemporal substitution" constraint. Special utility functions like "Epstein-Zin" utility incorporate both investor time preferences and consumption elasticity.

Portfolio Planning Issues	Expression in Academic Language
Asset Allocation	Long-term exposures to factor risks/systematic risks/priced risks
Threshold Income Requirements	Elasticity of intertemporat substitution (Extent to which an investor can tolerate consumption changes over time)
Preferred Portfolio Payoffs	Linear, Convex or Concave Asset Management Payoff Function
Subjective Risk Aversion	Subjective Discount Factor (Extent to which an investor can tolerate consumption variance among various economic states)
Utility of Wealth	Investor risk tolerance/risk aversion functions

FIGURE 7-5

Implications for Investment Policy

The single most challenging task in portfolio design and management is syncing the portfolio to an investor's preferences and constraints. In many cases, the asset management task requires a simultaneous solution to multiple variables under critical bound conditions. **FIGURE 7-5** provides some intuition regarding planning topics and their expression in the jargon of financial economics.

The single most challenging task in portfolio design and management is syncing the portfolio to an investor's preferences and constraints. In many cases, the asset management task requires a simultaneous solution to multiple variables under critical bound conditions.

The inaccessibility or difficulty of academic literature on investor utility and portfolio design[19] often means that investors and their advisors take little notice of topics beyond the portfolio asset allocation decision. Sometimes investors are asked to complete questionnaires regarding their investment goals ["income," "growth and income," "aggressive growth," "balanced," etc.] or about their preferences ["conservative," "moderate," "aggressive," etc.]. Although these questionnaires are ubiquitous on the internet, there is scant evidence to suggest that they produce useful results. One prominent vendor of financial services represents that answering seven multiple choice questions enables their portfolio selection

[19] Usually, the topic of "utility" occurs primarily at the Ph.D. in finance course level. See, for example, Ingersoll, Jonathan E., Theory of Financial Decision Making (Rowman and Littlefield, 1987) used in the Yale Ph.D. program for many years. The book begins with an extensive treatment of various utility functions and their underlying mathematics.

algorithm to (1) determine an investor's risk profile; and (2) provide an asset allocation well suited to it. In the vendor's defense, however, there follows a host of disclaimer statements suggesting that the recommendations may or may not be appropriate for any specific investor.

In addition to the problem of determining an investor's utility of wealth function, there appears to be no bullet-proof approach to asset management capable of guaranteeing success under all market conditions. Historically, a blind adherence to a Buy-and-Hold, a Constant Mix or a Floor + Multiplier asset management approach might have jeopardized critical financial objectives, especially in the presence of periodic distributions. These observations suggest a greater role in prudent asset management for ongoing portfolio monitoring and supervision throughout the portfolio's planning horizon. An enhanced role for monitoring the portfolio by measuring its likelihood for successfully accomplishing investor objectives implies that the traditional IPS document must also evolve.

Conceptually, transitioning from an IPS considered as an architectural blueprint to an IPS considered as a systems engineering process involves a two step process. Financial management comprises (1) portfolio design and implementation issues (the traditional asset allocation function of the IPS); and, (2) the set of future decisions that will help the portfolio evolve in a manner well suited to attain a possibly stochastic set of economic objectives. The Buy-and-Hold investor, for example, may wish to recalibrate portfolio risk and reward by readjusting the portfolio's asset allocation following a sustained period of equity drift. The Constant Mix investor may wish to refrain from restoring the full amount of exposure to risky assets lest a continuation of a high volatility regime increase the likelihood of penetrating a minimum wealth level. The Floor + Equity Multiplier investor may wish to raise the floor value to protect equity gains achieved during bull market environments.

Dynamic IPS provisions allow the investor to adjust the portfolio so it adapts to evolving conditions in way that best enables the finite sum of wealth to meet the investor's expectations and objectives. The indispensable tool for designing a dynamic IPS is advanced portfolio risk modeling capabilities that enable investors to "test drive" the economic consequences of a variety of asset management options prior to their implementation. Chapter Nine explores this topic further.

There is complete agreement that the suitability of an investment management approach depends on the investor's risk profile. A static IPS fixes investment decision making at the outset. This type of IPS may be appropriate for investors exhibiting certain common attitudes towards risk and reward, including Constant Relative Risk Aversion. If, however, the investor manifests risk/reward preferences that cannot be well characterized by a CRRA function, then adherence to a static IPS may not be feasible. An investor with above average sensitivity to changes in wealth may become too impatient with the rate of wealth growth in bull market regimes; or, too frustrated with the rate of wealth loss in bear market regimes. By contrast, a dynamic IPS has supervision and monitoring protocols that demand investors pay attention to recent market conditions and current uncertainty; and, by analyzing their current economic circumstances, to make prudent asset management elections. The criteria best suited for specifying the Investment Policy Statement's asset management approach are based on utility/preference metrics.